# NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

## TECHNICAL NOTE 2715

## THE THEORETICAL CHARACTERISTICS OF TRIANGULAR-TIP CONTROL SURFACES AT SUPERSONIC SPEEDS

## MACH LINES BEHIND TRAILING EDGES

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## SUMMARY

By means of linearized theory, generalized expressions in closed form have been obtained for the characteristics due to control-surface deflection $\left(C_{L_\delta},\ C_{l_\delta},\ C_{m_\delta},\text{ and } C_{h_\delta}\right)$ and due to wing angle of attack $\left(C_{h_\alpha}\right)$ for wing plan forms having triangular-tip control surfaces at supersonic speeds. The analysis considers wing trailing-edge sweep, control-surface geometry, and Mach number for the deflection characteristics. For $C_{h_\alpha}$ the effects of wing leading-edge sweep and aspect ratio are also included. The analysis is limited to configurations where the trailing edges are supersonic and where the innermost Mach line from the leading edge of the control-surface root chord does not intersect the wing root chord.

Charts are presented for the deflection characteristics for configurations having the sweep of the wing and control-surface trailing edges equal and for the location of the hinge line for balanced controls. Tables of calculations are given for configurations in which the wing and flap trailing edges are unequally swept.

The results of the analysis show that the effects of wing trailing-edge sweep on $C_{L_\delta}$, $C_{l_\delta}$, and $C_{m_\delta}$ are more significant than the effects of control-surface trailing-edge sweep $\left(C_{h_\delta}\right.$ is not affected by the wing trailing-edge sweep$\left.\right)$. Aileron reversal may occur for certain conditions when the control-surface leading edges are subsonic. Maximum rolling moments are obtained for control surfaces having supersonic leading edges.

## INTRODUCTION

By means of linearized theory, generalized expressions have been developed for the stability derivatives of a wide variety of wing

configurations at supersonic speeds (references 1 to 10). With the rapid development of supersonic aircraft and missile configurations, much interest has been focused on the problem of control. Several papers have been published (references 11 to 14) which present expressions for the theoretical control-surface characteristics of specific wing - control-surface configurations. By means of linearized theory, the present analysis extends these investigations by considering the effects of Mach number and geometry on the theoretical supersonic characteristics of a wide variety of configurations having triangular-tip control surfaces.

The present paper considers both subsonic and supersonic wing and control-surface leading edges but is restricted to cases in which the trailing edges of the wing and control surface are supersonic. Attention is given to configurations where the control-surface and wing trailing edges are swept both equal amounts (hereinafter referred to as the basic configuration) and unequal amounts. In addition to the inherent limitations of the linearized theory, the assumption is made that the innermost Mach line from the control-surface leading-edge apex does not intersect the root chord of the wing. The results are applicable to configurations having sweptforward or sweptback leading and trailing edges.

The results are presented in the form of equations for the evaluation of $C_{L_\delta}$, $C_{l_\delta}$, $C_{m_\delta}$, and $C_{h_\delta}$, charts for the rapid estimation of these quantities for the basic configurations, and tabulated calculations for configurations having unequally swept trailing edges. In addition, generalized equations are given for the evaluation of $C_{h_\alpha}$. A specific illustration is presented for a wing configuration having equally swept subsonic wing and control-surface leading edges.

## SYMBOLS

Free-stream conditions:

| | |
|---|---|
| $V$ | velocity |
| $M$ | Mach number |
| $\mu$ | Mach angle $\left(\sin^{-1}\frac{1}{M}\right)$ |
| $\beta = \sqrt{M^2 - 1}$ | |
| $\rho$ | mass density of air |

| | |
|---|---|
| $q$ | dynamic pressure $\left(\frac{1}{2}\rho V^2\right)$ |

Wing geometry:

| | |
|---|---|
| $b/2$ | semispan including flap span $(h_1 + b_f)$ |
| $h_1$ | distance from wing root chord to control-surface root chord (considered as wing semispan) |
| $c_r$ | root chord |
| $\Lambda$ | leading-edge sweep, degrees |
| $\Lambda_3$ | trailing-edge sweep, degrees |
| $m = \cot \Lambda$ | |
| $m_3 = \cot \Lambda_3$ | |
| $\alpha$ | angle of attack in stream direction, radians |

Control-surface geometry:

| | |
|---|---|
| $b_f$ | span |
| $c_{f_r}$ | root chord |
| $\Lambda_1$ | leading-edge sweep, degrees |
| $\Lambda_2$ | trailing-edge sweep, degrees |
| $\Lambda_{TE}$ | trailing-edge sweep of basic configuration, degrees $\left(\Lambda_{TE} = \Lambda_2 = \Lambda_3\right)$ |
| $m_1 = \cot \Lambda_1$ | |
| $m_2 = \cot \Lambda_2$ | |
| $m_{TE} = \cot \Lambda_{TE}$ | |
| $S_f$ | area |
| $A_f$ | aspect ratio $\left(b_f^2/S_f\right)$ |
| $\delta$ | deflection angle in stream direction, radians |
| $x_h$ | hinge-line location |

$x_h' = \dfrac{x_h}{c_{f_r}}$

| | |
|---|---|
| $(x_h')_B$ | $x_h'$ for balanced control surface |
| $\overline{M}$ | arbitrary moment of area of control surface |

Parameters used in analysis:

| | |
|---|---|
| $x, y$ | Cartesian coordinates of system of axes with origin at leading edge of wing root chord |
| $x_a, y_a$ | Cartesian coordinates of system of axes with origin at leading edge of control-surface root chord |
| $\overline{x}, \overline{y}$ | coordinates of center of lift of infinitesimal lift triangle |
| $\overline{x}'$ | normal distance from center of lift of infinitesimal lift triangle to hinge line $(\overline{x} - x_h)$ |

$t = \dfrac{\beta y}{x}$

$t_a = \dfrac{\beta y_a}{x_a}$

| | |
|---|---|
| $S'$ | area of integration |
| $\phi(x,y)$ | velocity potential |

Forces and moments:

| | |
|---|---|
| $L$ | lift |
| $L'$ | rolling moment |
| $M'$ | pitching moment |
| $H$ | hinge moment |
| $C_L$ | lift coefficient $(L/qS_f)$ |
| $C_l$ | rolling-moment coefficient $(L'/qb_fS_f)$ |
| $C_m$ | pitching-moment coefficient $(M'/qc_{f_r}S_f)$ |

$C_h$ hinge-moment coefficient $\left(9H/2qb_fc_{f_r}{}^2\right)$

$\left.\begin{matrix}(C_{L_\delta})_f \\ (C_{L_\alpha})_f\end{matrix}\right\}$ lift-coefficient derivative excluding lift induced on wing by control surface

$\Delta p$ pressure differential existing across surfaces of flat plate

Formula parameters:

$\left.\begin{matrix}A_1, A_2, \ldots, A_5 \\ B_1, B_2, \ldots, B_9 \\ k_1, k_2 \\ \gamma_1, \gamma_2 \\ R \\ \nu\end{matrix}\right\}$ functions of wing and control-surface geometry, defined where used

$E\left(\sqrt{1 - m^2\beta^2}\right)$ complete elliptic integral of second kind with modulus $k = \sqrt{1 - m^2\beta^2}$, $\left(\int_0^{\pi/2} \sqrt{1 - k^2\sin^2 z}\, dz\right)$

Subscripts:

$\delta$ and $\alpha$ refer to partial derivatives of each coefficient with respect to $\delta$ and $\alpha$, respectively; for example, $C_{h_\alpha} = \frac{\partial}{\partial\alpha}(C_h)$

f refers to control surface

0 refers to an origin located at root-chord leading edge of control surface

# ANALYSIS

## Scope

The present analysis, based on the solution of the linearized equation for steady supersonic flow, employs the methods of references 15 to 20 to derive expressions for the control-surface characteristics due to deflection $C_{L_\delta}$, $C_{l_\delta}$, $C_{m_\delta}$, and $C_{h_\delta}$ and due to wing angle of attack $C_{h_\alpha}$. This investigation includes both subsonic and supersonic leading edges (fig. 1) but is restricted to wing - control-surface configurations having supersonic trailing edges. In addition to the fundamental limitations of the linearized theory, the assumption is made that the innermost Mach line emanating from the root-chord leading-edge apex of the control surface does not intersect the root chord of the wing (fig. 1). Furthermore, the gap between the deflected control surface and the wing is assumed to be completely free of leakage (thus effects beyond the scope of linear theory are not considered; see reference 19).

## Basic Considerations

The deflection characteristics $\left(C_{L_\delta},\ C_{l_\delta},\ C_{m_\delta},\text{ and } C_{h_\delta}\right)$ and the characteristic due to wing angle of attack $\left(C_{h_\alpha}\right)$ can be derived when the respective lifting-pressure distributions are known. The linearized lifting-pressure-coefficient distribution is given by

$$\frac{\Delta p}{q} = \frac{4}{V} \frac{\partial}{\partial x}\left[\phi(x,y)\right] \tag{1}$$

where $\phi(x,y)$ is the perturbation velocity potential on the upper surface of the control surface. This potential due to control-surface deflection may be determined by the use of references 15 to 17 or may be obtained directly from references 19 and 20. As a consequence of the use of linearized theory, the wing is treated as being at zero angle of attack and, hence, the pressure distributions for the deflected control surface (table I) are independent of wing geometry.

The pressure distributions due to angle of attack in the vicinity of the wing tip are derived for configurations in which the control-surface and wing leading edges are unequally swept (fig. 1). These pressure distributions (table II) have been derived by means of references 15 to 18 for both subsonic and supersonic leading edges. The derivations of the pressure distributions for the configurations having supersonic wing and control-surface leading edges are straightforward; whereas

for the configurations having subsonic wing and control-surface leading edges, an approximation (in which the effects of the secondary Mach reflections are essentially neglected, reference 18) to the exact linearized tip velocity potential is applied. This approximation reduces the integration of the disturbances in the forecone (OBPE) of a point x,y (fig. 2) to an integration over the parallelogram (APCD). Evvard (reference 18) has shown that, for certain limiting cases, the parallelogram representation reduces to known exact linear solutions; however, it is approximate for the cases where the Mach lines lie ahead of the leading edges. As the angle between the Mach lines and the leading edges decreases $\left(m \rightarrow \frac{1}{\beta}\right)$, the error in the approximation approaches zero. More detailed information regarding the approximation may be found in reference 21.

Evvard (reference 18) compared the pressure distributions on a thin pointed wing derived by the approximate and exact linear solutions for the velocity potential and observed that they differed only in the constant of proportionality. The error between the constant $K = \frac{4}{\pi(1 + m\beta)}$ of the approximate linearized solution and $\frac{1}{E\left(\sqrt{1 - m^2\beta^2}\right)}$ of the exact linearized solution increases from 0 to about 27 percent as the component of Mach number normal to the leading edge $(m\beta)$ decreases from 1 to 0. When the approximate solution for the velocity potential is modified by replacing $K$ by $\frac{1}{E\left(\sqrt{1 - m^2\beta^2}\right)}$, the result reduces to the exact linearized solution for the limiting case $m = m_1$ $\left(\text{note also that } K \text{ and } \frac{1}{E\left(\sqrt{1 - m^2\beta^2}\right)} \text{ are identical at } m = \frac{1}{\beta}\right)$. Although there is no guarantee that such good agreement may be expected for all conditions $(m_1 \neq m)$, the modified solution will be used herein to obtain $C_{h_\alpha}$. For a more thorough discussion regarding the constant of proportionality, see reference 18.

### Derivation of $C_{L_\delta}$, $C_{l_\delta}$, $C_{m_\delta}$, $C_{h_\delta}$, and $C_{h_\alpha}$

The general forms of the expressions for the characteristics due to control-surface deflection and wing angle of attack are

$$C_{L_\delta} = \frac{1}{S_f} \int_{S'} \int \frac{\Delta p}{q\delta} \, dS' \tag{2}$$

$$C_{l_\delta} = \frac{1}{b_f S_f} \int_{S'} \int \bar{y} \frac{\Delta p}{q\delta} \, dS' \tag{3}$$

$$C_{m_\delta} = - \frac{1}{c_{f_r} S_f} \int_{S'} \int \bar{x} \frac{\Delta p}{q\delta} \, dS' \tag{4}$$

$$C_{h_\delta} = - \frac{9}{2 b_f {c_{f_r}}^2} \int_{S_f} \int \bar{x}' \frac{\Delta p}{q\delta} \, dS' \tag{5}$$

$$C_{h_\alpha} = - \frac{9}{2 b_f {c_{f_r}}^2} \int_{S_f} \int \bar{x}' \frac{\Delta p}{q\alpha} \, dS' \tag{6}$$

where the integration includes the region of induced lift in addition to the control-surface area for equations (2) to (4) and includes only the control-surface area in equations (5) and (6). The factor by which the hinge-moment coefficients were made nondimensional includes the root-mean-square chord of the control surface.

Since the pressure-coefficient distribution due to control-surface deflection $\Delta p/q$ is conical (see table I), the integrations indicated by equations (2) to (5) may be conveniently accomplished by means of polar integration. The details and limits for polar integration are presented in figure 3, and the resulting generalized equations and specific cases are presented in appendix A for the subsonic and sonic leading edges and in appendix B for the supersonic leading edges.

The form of the pressure coefficient due to wing angle of attack $\Delta p/q$ is conical for the supersonic leading edges (table II), and hence, polar integration is also applicable in this case. (See appendix C.) However, due to the nonconical form of $\Delta p/q$ in the tip region for the subsonic leading edges (table II), integration is performed in Cartesian coordinates in appendix D.

## Control-Surface Parameters

The rolling-moment- and pitching-moment-coefficient derivatives have been obtained about the axes of a Cartesian coordinate system located at the leading edge of the control-surface root chord (fig. 3) and may be transferred to any convenient reference axes by means of

$$C_{l_\delta}{}^* = C_{l_\delta} + \frac{Y}{b_f} C_{L_\delta} \tag{7}$$

$$C_{m_\delta}{}^* = C_{m_\delta} + \frac{2X}{c_{f_r}} C_{L_\delta} \tag{8}$$

where the asterisked quantities in equations (7) and (8) refer to the transferred quantities and X and Y represent the horizontal and vertical distances between the present axes and the desired ones.

The hinge-moment coefficients were made nondimensional by dividing the hinge moment by the moment of area of the entire control surface which incorporated the control-surface root-mean-square chord. If these coefficients are to be referenced about another moment of area, then the coefficients given herein are multiplied by the ratio $\frac{2b_f c_{f_r}{}^2}{9\bar{M}}$ where $\bar{M}$ is the new moment of area.

Since the hinge-moment coefficients are linearly dependent upon the nondimensional hinge-line location $x_h'$, the results are presented in terms of the hinge-moment parameter about the origin (independent of $x_h'$) $\left(C_{h_{\alpha,\delta}}\right)_0$ and the moment of lift[1] on the control surface $\left(C_{L_{\alpha,\delta}}\right)_f$, where $C_{h_{\alpha,\delta}}$ means $C_{h_\alpha}$ or $C_{h_\delta}$ and $\left(C_{L_{\alpha,\delta}}\right)_f$ means $\left(C_{L_\alpha}\right)_f$ or $\left(C_{L_\delta}\right)_f$. The assembled hinge-moment coefficient is

$$C_{h_{\alpha,\delta}} = \left(C_{h_{\alpha,\delta}}\right)_0 + x_h'\left(C_{L_{\alpha,\delta}}\right)_f \tag{9}$$

For a balanced control surface, $C_{h_{\alpha,\delta}}$ must be zero and hence the location of the hinge line for this condition is

$$\left(x_h'\right)_B = -\frac{\left(C_{h_{\alpha,\delta}}\right)_0}{\left(C_{L_{\alpha,\delta}}\right)_f} \tag{10}$$

[1]The lift referred to includes only the lift on the control surface and excludes the lift induced on the wing by the control surface.

It should not be assumed from equation (10) that the balanced hinge-line location is identical for control-surface deflection and wing angle of attack. On the contrary, this condition is difficult to incorporate in the design by virtue of the many geometrical variables affecting the quantities on the right-hand side of equation (10). However, the location of the balanced hinge line for a configuration in which the wing is at an angle of attack $\alpha$ and the control surface at an angle $\alpha + \delta$ may be obtained by applying equation (9) for $\delta$ and for $\alpha$ and writing the results in the form

$$C_h = \left[\left(C_{h_\delta}\right)_0 + \left(C_{L_\delta}\right)_f x_h^*\right]\delta \tag{11}$$

and

$$C_h = \left[\left(C_{h_\alpha}\right)_0 + \left(C_{L_\alpha}\right)_f x_h^*\right]\alpha \tag{12}$$

where $x_h^*$ is the desired balance location. If equations (11) and (12) are added, the resulting expression gives the hinge-moment coefficient of a plan form in which the wing section is at an angle of attack $\alpha$ and the control is deflected at an angle of $\alpha + \delta$. If this result is set equal to zero (that is, the new configuration is balanced),

$$x_h^* = \frac{\left(C_{h_\delta}\right)_0 + \left(C_{h_\alpha}\right)_0 \frac{\alpha}{\delta}}{\left(C_{L_\delta}\right)_f + \left(C_{L_\alpha}\right)_f \frac{\alpha}{\delta}} \tag{13}$$

When $\alpha/\delta$ is zero, equation (13) yields the hinge-line balance position for deflection obtainable from equation (10), and, when $\alpha/\delta$ is infinite, the corresponding position for angle of attack is obtained.

## RESULTS AND DISCUSSION

Appendixes A and B present the details and resulting expressions for the control-surface deflection characteristics for configurations having subsonic and supersonic leading edges, respectively. The formula numbers from appendixes A and B for the expressions developed from equations (2) to (6) corresponding to every possible combination of the control-surface parameters ($m_1\beta$ and $m_2\beta$) and the wing parameter ($m_3\beta$) within the scope of the present paper are presented in table III for convenient reference.

The details and formulas for $C_{h_\alpha}$ are presented in appendixes C and D for supersonic and subsonic leading edges, respectively. These results include the effects of wing leading-edge sweep and aspect ratio as well as Mach number and control-surface geometry. In addition, an illustrative calculation is presented in appendix D for the case where the control-surface and wing leading-edge sweeps are equal.

Design charts are presented for the deflection characteristics for the basic configuration (a configuration having equally swept wing and control-surface trailing edges) in figures 4 to 8. Figure 9 is a plot of the location of the hinge lines for balanced deflected control surfaces. Calculations for configurations in which the wing and control-surface trailing edges are unequally swept are given in table IV.

Analysis of the calculations presented in table IV for deflected controls when either the wing or control-surface trailing-edge sweep is held fixed while the other is varied indicates that more significant changes in $C_{L_\delta}$, $C_{l_\delta}$, and $C_{m_\delta}$ from the basic values occur for variations in wing trailing-edge sweep than for similar variations in the control-surface trailing-edge sweep.

A study of figures 4 and 5 suggests the use of slightly swept supersonic control-surface leading edges in order to obtain greater lift and rolling-moment coefficients. When structural requirements demand the location of the hinge-line balance position (for deflection) in the vicinity of the midchord, the control-surface trailing edges should be either unswept or slightly swept forward and the leading edges should be swept back (fig. 9(b)). Generally the center of pressure of the hinge forces for angle of attack is to the rear of the position for the deflection-alone case. The value of $(x_h')_B$ (fig. 9) for deflection may be moved rearward by decreasing the sweep of the control-surface trailing edge or increasing that of the control-surface leading edge, the latter being less critical than the former. Due to the many variables (wing and control-surface geometry) affecting $(x_h')_B$ for angle of attack, it is suggested that, for a specific configuration, the effect of any change in the control-surface geometry on $(x_h')_B$ be investigated and the result compared with the value for deflection. However, it is pointed out that for a configuration having supersonic wing and control-surface leading edges where the Mach wave from the wing root-chord apex does not intersect the wing tip, the value of $(x_h')_B$ for angle of attack is merely a function of the sweep of the wing and control-surface leading edges and the control-surface trailing edge. For this case, a short trial-and-error method of obtaining a better value of the wing leading-edge sweep than the given value may be used to obtain a value of $(x_h')_B$ for angle of attack which may be more satisfactory.

Another reason for choosing supersonic control-surface leading edges is to prevent the possibility of aileron reversal (see fig. 5) inherent under certain conditions for subsonic leading edges.

In the preceding discussion the role of the wing trailing-edge sweep in the design of controls was not specifically mentioned. Since the results in figures 4 to 8 are for basic configurations, the wing trailing-edge sweep $m_3\beta$ had the same value as the control-surface trailing-edge sweep $m_2\beta$. In an actual design problem where the wing trailing-edge sweep is known, a basic configuration is not necessarily desired since this restriction could fix the control-surface leading-edge sweep at some undesirable value. However, if the wing trailing edge is swept forward slightly, such a basic configuration probably would be desired. If the basic configuration is not the desired one, design would proceed as follows (consider $\beta = 1$):

(1) For low values of sweepforward of the control-surface trailing edge $m_2$, $C_{l_\delta}$ does not vary much with supersonic leading-edge sweep. However, a few investigatory calculations with different values of $m_2$ and $m_1$ (constant $m_3$) may be desirable, where

$$m_1 = \frac{1}{\frac{1}{m_2} + \frac{2}{A_f}} = \frac{1}{\frac{1}{m_2} + \frac{2c_{f_r}}{b_f}}$$

(2) Having obtained the control-surface geometry (wing geometry known), the locations for balanced hinge lines $(x_h')_B$ for deflection and for angle of attack are obtained from equation (10). If these values are acceptable, no problem is anticipated in locating the hinge line. However, if the travel of the hinge-line balance is greater than desired, the method of reducing the travel should be dictated by the positions of $(x_h')_B$ for deflection and $(x_h')_B$ for angle of attack. If the value of $(x_h')_B$ for deflection is more forward than desired, it may be moved rearward as previously described, and if the value of $(x_h')_B$ for angle of attack is unreasonably rearward, a systematic trial-and-error method involving wing geometry is suggested.

## CONCLUSIONS

By means of linear theory, generalized expressions in closed form have been obtained for the characteristics due to control-surface

deflection $\left(C_{L_\delta},\ C_{l_\delta},\ C_{m_\delta},\ \text{and}\ C_{h_\delta}\right)$ and due to wing angle of attack $\left(C_{h_\alpha}\right)$ for wing plan forms having triangular-tip control surfaces at supersonic speeds.

Charts are presented for the deflection characteristics for basic configurations having the sweep of the wing and control-surface trailing edges equal and for the balanced control-surface hinge-line locations. Tables of calculations are given for configurations in which the wing and control-surface trailing edges are unequally swept. From these results the following conclusions are made:

1. The results of the analysis indicate that characteristics are not greatly affected by varying either the wing or the control-surface trailing-edge sweep; however, a change in sweep of the wing trailing edge will affect $C_{L_\delta}$, $C_{l_\delta}$, and $C_{m_\delta}$ more significantly than a corresponding change in the control-surface trailing edge $\left(C_{h_\delta}\right.$ is not affected by the wing trailing-edge sweep$\left.\right)$.

2. The leading edges of the control surfaces should be supersonic in order to obtain the largest possible rolling moments and to prevent the possibility of aileron reversal inherent under certain conditions for configurations having subsonic leading edges.

3. Unswept or slightly sweptforward control-surface trailing edges combined with sweptback leading edges are suggested if the hinge-line balance position for control deflection is to be in the midchord vicinity of the control-surface root chord.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., January 25, 1952

# APPENDIX A

## GENERALIZED AND SPECIFIC FORMULAS FOR $C_{L_\delta}$, $C_{l_\delta}$, $C_{m_\delta}$, AND $C_{h_\delta}$ FOR SUBSONIC AND SONIC LEADING EDGES

The generalized formulas for the derivation of the characteristics $C_{L_\delta}$, $C_{l_\delta}$, $C_{m_\delta}$, and $C_{h_\delta}$ for subsonic and sonic leading edges are:

$$C_{L_\delta} = \frac{8\sqrt{m_1\beta}\left(m_2 - m_1\right)\beta}{\pi m_2\left(1 + m_1\beta\right)}\left[m_2^2 \int_{t_a=0}^{t_a=m_1\beta} \sqrt{\frac{1 + t_a}{m_1\beta - t_a}} \frac{dt_a}{\left(m_2\beta - t_a\right)^2} + m_3^2 \int_{t_a=-1}^{t_a=0} \sqrt{\frac{1 + t_a}{m_1\beta - t_a}} \frac{dt_a}{\left(m_3\beta - t_a\right)^2}\right] \tag{A1}$$

$$C_{l_\delta} = \frac{16\left(m_2 - m_1\right)^2\beta^2}{3\pi m_2^2\sqrt{m_1\beta}\left(1 + m_1\beta\right)}\left[m_2^3 \int_0^{m_1\beta} \sqrt{\frac{1 + t_a}{m_1\beta - t_a}} \frac{t_a\, dt_a}{\left(m_2\beta - t_a\right)^3} + m_3^3 \int_{-1}^{0} \sqrt{\frac{1 + t_a}{m_1\beta - t_a}} \frac{t_a\, dt_a}{\left(m_3\beta - t_a\right)^3}\right] \tag{A2}$$

$$C_{m_\delta} = -\frac{16\sqrt{m_1\beta}\left(m_2 - m_1\right)\beta^2}{3\pi m_2\left(1 + m_1\beta\right)}\left[m_2^3 \int_0^{m_1\beta} \sqrt{\frac{1 + t_a}{m_1\beta - t_a}} \frac{dt_a}{\left(m_2\beta - t_a\right)^3} + m_3^3 \int_{-1}^{0} \sqrt{\frac{1 + t_a}{m_1\beta - t_a}} \frac{dt_a}{\left(m_3\beta - t_a\right)^3}\right] \tag{A3}$$

$$C_{h_\delta} = -\frac{6\sqrt{m_1\beta}m_2(m_2 - m_1)\beta}{\pi(1 + m_1\beta)}\left[2m_2\beta\int_0^{m_1\beta}\sqrt{\frac{1 + t_a}{m_1\beta - t_a}}\frac{dt_a}{(m_2\beta - t_a)^3} - \right.$$

$$\left. 3x_h'\int_0^{m_1}\sqrt{\frac{1 + t_a}{m_1\beta - t_a}}\frac{dt_a}{(m_2\beta - t_a)^2}\right] \tag{A4}$$

For the basic configuration $(m_2 = m_3 = m_{TE})$, the formulas for the characteristics resulting from equations (A1) to (A4) are:

$$C_{L_\delta} = \frac{4\sqrt{m_1}m_{TE}}{\sqrt{(m_{TE} - m_1)(m_{TE}\beta + 1)}} \tag{A5}$$

$$C_{l_\delta} = \frac{2m_{TE}(3m_1m_{TE}\beta + 4m_1 - m_{TE})}{3(m_{TE}\beta + 1)\sqrt{(m_{TE} - m_1)(m_{TE}\beta + 1)m_1}} \tag{A6}$$

$$C_{m_\delta} = -\frac{2\sqrt{m_1}m_{TE}^2(4m_{TE}\beta - m_1\beta + 3)}{3\left[(m_{TE} - m_1)(m_{TE}\beta + 1)\right]^{3/2}} \tag{A7}$$

$$C_{h_\delta} = (C_{h_\delta})_0 + x_h'(C_{L_\delta})_f \tag{A8}$$

$$(C_{h_\delta})_0 = -\frac{3m_1m_{TE}^2}{\pi(1 + m_1\beta)(m_{TE}\beta + 1)(m_1 - m_{TE})}\left[\frac{m_1m_{TE}\beta + 2m_1 - 4m_{TE}^2\beta - 5m_{TE}}{m_{TE}^2} + \right.$$

$$\left.\frac{(1 + m_1\beta)(m_1\beta - 4m_{TE}\beta - 3)A_1}{2\sqrt{(m_{TE} - m_1)(m_{TE}\beta + 1)m_1}}\right] \tag{A9}$$

where

$$A_1 = \frac{\pi}{2} + \tan^{-1}\left[\frac{m_1 m_{TE}\beta + 2m_1 - m_{TE}}{2\sqrt{(m_{TE} - m_1)(m_{TE}\beta + 1)m_1}}\right] \tag{A10}$$

$$\left(C_{L_\delta}\right)_f = \frac{18m_1 m_{TE}}{\pi(1 + m_1\beta)}\left[\frac{1}{m_{TE}} + \frac{(1 + m_1\beta)A_1}{2\sqrt{(m_{TE} - m_1)(m_{TE}\beta + 1)m_1}}\right] \tag{A11}$$

For configurations having $1 < |m_2\beta| < \infty$ and $1 < |m_3\beta| < \infty$, $1 < |m_2\beta| < \infty$ and $m_3\beta = 1$, $m_2\beta = 1$ and $1 < |m_3\beta| < \infty$, or $m_2\beta = 1$ and $m_3\beta = 1$, the formulas for the characteristics resulting from equations (A1) to (A4) are:

$$C_{L_\delta} = \frac{8m_1(m_1 - m_2)}{\pi m_2(1 + m_1\beta)}\left\{\frac{m_2^2}{m_1 - m_2}\left[\frac{1}{m_2} + \frac{(1 + m_1\beta)A_2}{2\sqrt{(m_2 - m_1)(m_2\beta + 1)m_1}}\right] - \frac{m_3^2}{m_1 - m_3}\left[\frac{1}{m_3} + \frac{(1 + m_1\beta)A_3}{2\sqrt{(m_3 - m_1)(m_3\beta + 1)m_1}}\right]\right\} \tag{A12}$$

where

$$A_2 = \frac{\pi}{2} + \tan^{-1}\left[\frac{m_1 m_2\beta + 2m_1 - m_2}{2\sqrt{(m_2 - m_1)(m_2\beta + 1)m_1}}\right] \tag{A13}$$

and

$$A_3 = \tan^{-1}\left[\frac{m_1 m_3\beta + 2m_1 - m_3}{2\sqrt{(m_3 - m_1)(m_3\beta + 1)m_1}}\right] - \frac{\pi}{2} \tag{A14}$$

$$C_{l_\delta} = \frac{4(m_1 - m_2)^2}{3\pi m_2^2(1 + m_1\beta)}\left\{\frac{m_2^3}{(m_1 - m_2)^2(m_2\beta + 1)}\left[\frac{3m_1m_2\beta + 2m_1 + m_2}{m_2} + \frac{(1 + m_1\beta)(3m_1m_2\beta + 4m_1 - m_2)A_2}{2\sqrt{(m_2 - m_1)(m_2\beta + 1)m_1}}\right] - \frac{m_3^3}{(m_1 - m_3)^2(m_3\beta + 1)}\left[\frac{3m_1m_3\beta + 2m_1 + m_3}{m_3} + \frac{(1 + m_1\beta)(3m_1m_3\beta + 4m_1 - m_3)A_3}{2\sqrt{(m_3 - m_1)(m_3\beta + 1)m_1}}\right]\right\} \tag{A15}$$

$$C_{m_\delta} = -\frac{4m_1(m_1 - m_2)}{3\pi m_2(1 + m_1\beta)}\left\{\frac{m_2^3}{(m_1 - m_2)^2(m_2\beta + 1)}\left[\frac{2m_1 + m_1m_2\beta - 4m_2^2\beta - 5m_2}{m_2^2} + \frac{(1 + m_1\beta)(m_1\beta - 4m_2\beta - 3)A_2}{2\sqrt{(m_2 - m_1)(m_2\beta + 1)m_1}}\right] - \frac{m_3^3}{(m_1 - m_3)^2(m_3\beta + 1)}\left[\frac{2m_1 + m_1m_3\beta - 4m_3^2\beta - 5m_3}{m_3^2} + \frac{(1 + m_1\beta)(m_1\beta - 4m_3\beta - 3)A_3}{2\sqrt{(m_3 - m_1)(m_3\beta + 1)m_1}}\right]\right\} \tag{A16}$$

$$\left(C_{h_\delta}\right)_0 = -\frac{3m_1m_2^2}{\pi\left(1 + m_1\beta\right)\left(m_2\beta + 1\right)\left(m_1 - m_2\right)}\left[\frac{2m_1 + m_1m_2\beta - 4m_2^2\beta - 5m_2}{m_2^2} + \right.$$

$$\left.\frac{\left(1 + m_1\beta\right)\left(m_1\beta - 4m_2\beta - 3\right)A_2}{2\sqrt{\left(m_2 - m_1\right)\left(m_2\beta + 1\right)m_1}}\right] \tag{A17}$$

$$\left(C_{L_\delta}\right)_f = \frac{18m_1m_2}{\pi\left(1 + m_1\beta\right)}\left[\frac{1}{m_2} + \frac{\left(1 + m_1\beta\right)A_2}{2\sqrt{\left(m_2 - m_1\right)\left(m_2\beta + 1\right)m_1}}\right] \tag{A18}$$

For configurations having $1 < |m_2\beta| < \infty$ and $m_3\beta = \infty$, the formulas for the characteristics resulting from equations (A1) to (A4) are:

$$C_{L_\delta} = \frac{8m_1\left(m_1 - m_2\right)}{\pi m_2\left(1 + m_1\beta\right)}\left\{\frac{m_2^2}{m_1 - m_2}\left[\frac{1}{m_2} + \frac{\left(1 + m_1\beta\right)A_2}{2\sqrt{\left(m_2 - m_1\right)\left(m_2\beta + 1\right)m_1}}\right] + \right.$$

$$\left. 1 + \frac{\left(1 + m_1\beta\right)A_4}{2\sqrt{m_1\beta}}\right\} \tag{A19}$$

where

$$A_4 = -2\tan^{-1}\left(\frac{1}{\sqrt{m_1\beta}}\right) \tag{A20}$$

$$C_{l_\delta} = \frac{4(m_1 - m_2)^2}{3\pi m_2^2(1 + m_1\beta)} \left\{ \frac{m_2^3}{(m_1 - m_2)^2(m_2\beta + 1)} \left[ \frac{3m_1m_2\beta + 2m_1 + m_2}{m_2} + \right. \right.$$

$$\left. \frac{(1 + m_1\beta)(3m_1m_2\beta + 4m_1 - m_2)A_2}{2\sqrt{(m_2 - m_1)(m_2\beta + 1)m_1}} \right] - \frac{1}{\beta} \left[ 3m_1\beta + 1 + \right.$$

$$\left. \left. \frac{(1 + m_1\beta)(3m_1\beta - 1)A_4}{2\sqrt{m_1\beta}} \right] \right\} \tag{A21}$$

$$C_{m_\delta} = -\frac{4m_1(m_1 - m_2)}{3\pi m_2(1 + m_1\beta)} \left\{ \frac{m_2^3}{(m_1 - m_2)^2(m_2\beta + 1)} \left[ \frac{2m_1 + m_1m_2\beta - 4m_2^2\beta - 5m_2}{m_2^2} + \right. \right.$$

$$\left. \left. \frac{(1 + m_1\beta)(m_1\beta - 4m_2\beta - 3)A_2}{2\sqrt{(m_2 - m_1)(m_2\beta + 1)m_1}} \right] + 2\left[ 2 + \frac{(1 + m_1\beta)A_4}{\sqrt{m_1\beta}} \right] \right\} \tag{A22}$$

$$\left(C_{h_\delta}\right)_0 = -\frac{3m_1m_2^2}{\pi(1 + m_1\beta)(m_2\beta + 1)(m_1 - m_2)} \left[ \frac{2m_1 + m_1m_2\beta - 4m_2^2\beta - 5m_2}{m_2^2} + \right.$$

$$\left. \frac{(1 + m_1\beta)(m_1\beta - 4m_2\beta - 3)A_2}{2\sqrt{(m_2 - m_1)(m_2\beta + 1)m_1}} \right] \tag{A23}$$

$$\left(C_{L_\delta}\right)_f = \frac{18m_1m_2}{\pi(1 + m_1\beta)} \left[ \frac{1}{m_2} + \frac{(1 + m_1\beta)A_2}{2\sqrt{(m_2 - m_1)(m_2\beta + 1)m_1}} \right] \tag{A24}$$

For configurations having $m_2\beta = \infty$ and $1 < |m_3\beta| < \infty$ or $m_2\beta = \infty$ and $m_3\beta = 1$, the formulas for the characteristics resulting from equations (A1) to (A4) are:

$$C_{L_\delta} = \frac{8m_1}{\pi(1 + m_1\beta)}\left\{1 + (1 + m_1\beta)(A_4 + \pi) + \frac{m_3^2}{m_1 - m_3}\left[\frac{1}{m_3} + \frac{(1 + m_1\beta)A_3}{2\sqrt{(m_3 - m_1)(m_3\beta + 1)m_1}}\right]\right\} \tag{A25}$$

$$C_{l_\delta} = \frac{4}{3\pi(1 + m_1\beta)}\left\{\frac{1}{\beta}\left[3m_1\beta + 1 + \frac{(1 + m_1\beta)(3m_1\beta - 1)(A_4 + \pi)}{2\sqrt{m_1\beta}}\right] - \frac{m_3^3}{(m_1 - m_3)^2(m_3\beta + 1)}\left[\frac{3m_1m_3\beta + 2m_1 + m_3}{m_3} + \frac{(1 + m_1\beta)(3m_1m_3\beta + 4m_1 - m_3)A_3}{2\sqrt{(m_3 - m_1)(m_3\beta + 1)m_1}}\right]\right\} \tag{A26}$$

$$C_{m_\delta} = -\frac{4m_1}{3\pi(1 + m_1\beta)}\left\{2\left[2 + \frac{(1 + m_1\beta)(A_4 + \pi)}{\sqrt{m_1\beta}}\right] + \frac{m_3^3}{(m_1 - m_3)^2(m_3\beta + 1)}\left[\frac{m_1m_3\beta + 2m_1 - 4m_3^2\beta - 5m_3}{m_3^2} + \frac{(1 + m_1\beta)(m_1\beta - 4m_3\beta - 3)A_3}{2\sqrt{(m_3 - m_1)(m_3\beta + 1)m_1}}\right]\right\} \tag{A27}$$

$$\left(C_{h_\delta}\right)_0 = -\frac{6m_1}{\pi\left(1 + m_1\beta\right)}\left[2 + \frac{\left(1 + m_1\beta\right)\left(A_4 + \pi\right)}{\sqrt{m_1\beta}}\right] \tag{A28}$$

$$\left(C_{L_\delta}\right)_f = -\frac{3}{2}\left(C_{h_\delta}\right)_0 \tag{A29}$$

For configurations having $m_2\beta = \infty$ and $m_3\beta = \infty$, the formulas for the characteristics resulting from equations (A1) to (A4) are:

$$C_{L_\delta} = 4\sqrt{\frac{m_1}{\beta}} \tag{A30}$$

$$C_{l_\delta} = \frac{2\left(3m_1\beta - 1\right)}{3\beta\sqrt{m_1\beta}} \tag{A31}$$

$$C_{m_\delta} = -\frac{8}{3}\sqrt{\frac{m_1}{\beta}} \tag{A32}$$

$$\left(C_{h_\delta}\right)_0 = -\frac{6m_1}{\pi\left(1 + m_1\beta\right)}\left[2 + \frac{\left(1 + m_1\beta\right)\left(A_4 + \pi\right)}{\sqrt{m_1\beta}}\right] \tag{A33}$$

$$\left(C_{L_\delta}\right)_f = -\frac{3}{2}\left(C_{h_\delta}\right)_0 \tag{A34}$$

For configurations having $m_2\beta = 1$ and $m_3\beta = \infty$, the formulas for the characteristics resulting from equations (A1) to (A4) are:

$$C_{L_\delta} = \frac{8m_1(1 - m_1\beta)}{\pi(1 + m_1\beta)}\left\{\frac{1}{1 - m_1\beta}\left[1 + \frac{(1 + m_1\beta)A_5}{2\sqrt{2m_1\beta(1 - m_1\beta)}}\right] - 1 - \frac{(1 + m_1\beta)A_4}{2\sqrt{m_1\beta}}\right\} \tag{A35}$$

where

$$A_5 = \frac{\pi}{2} + \tan^{-1}\left[\frac{3m_1\beta - 1}{2\sqrt{2m_1\beta(1 - m_1\beta)}}\right] \tag{A36}$$

$$C_{l_\delta} = \frac{4(m_1\beta - 1)^2}{3\pi\beta(1 + m_1\beta)}\left\{\frac{1}{2(m_1\beta - 1)^2}\left[5m_1\beta + 1 + \frac{(1 + m_1\beta)(7m_1\beta - 1)A_5}{2\sqrt{2m_1\beta(1 - m_1\beta)}}\right] - 3m_1\beta - 1 - \frac{(1 + m_1\beta)(3m_1\beta - 1)A_4}{2\sqrt{m_1\beta}}\right\} \tag{A37}$$

$$C_{m_\delta} = -\frac{4m_1(1 - m_1\beta)}{3\pi(1 + m_1\beta)}\left\{\frac{1}{2(1 - m_1\beta)^2}\left[3(3 - m_1\beta) + \frac{(1 + m_1\beta)(7 - m_1\beta)A_5}{2\sqrt{2m_1\beta(1 - m_1\beta)}}\right] - 2\left[2 + \frac{(1 + m_1\beta)A_4}{\sqrt{m_1\beta}}\right]\right\} \tag{A38}$$

$$\left(C_{h_\delta}\right)_0 = -\frac{3m_1}{2\pi(1 - m_1^2\beta^2)}\left[3(3 - m_1\beta) + \frac{(1 + m_1\beta)(7 - m_1\beta)A_5}{2\sqrt{2m_1\beta(1 - m_1\beta)}}\right] \tag{A39}$$

$$\left(C_{L_\delta}\right)_f = \frac{18m_1}{\pi(1 + m_1\beta)}\left[1 + \frac{(1 + m_1\beta)A_5}{2\sqrt{2m_1\beta(1 - m_1\beta)}}\right] \tag{A40}$$

For configurations having $m_2\beta = -1$ and $1 < |m_3\beta| < \infty$ or $m_2\beta = -1$ and $m_3\beta = 1$, the formulas for the characteristics resulting from equations (A1) to (A4) are:

$$C_{L_\delta} = \frac{8m_1}{\pi}\left\{\frac{2}{1 + m_1\beta} + \frac{{m_3}^2}{m_1 - m_3}\left[\frac{1}{m_3} + \frac{(1 + m_1\beta)A_3}{2\sqrt{(m_3 - m_1)(m_3\beta + 1)m_1}}\right]\right\} \tag{A41}$$

$$C_{l_\delta} = \frac{4(1 + m_1\beta)}{3\pi}\left\{\frac{16m_1}{3(1 + m_1\beta)^2} - \frac{{m_3}^3}{(m_1 - m_3)^2(m_3\beta + 1)}\left[\frac{3m_1m_3\beta + 2m_1 + m_3}{m_3} + \frac{(1 + m_1\beta)(3m_1m_3\beta + 4m_1 - m_3)A_3}{2\sqrt{(m_3 - m_1)(m_3\beta + 1)m_1}}\right]\right\} \tag{A42}$$

$$C_{m_\delta} = -\frac{4m_1}{3\pi}\left\{\frac{8(m_1\beta + 3)}{3(1 + m_1\beta)^2} + \frac{{m_3}^3}{(m_1 - m_3)^2(m_3\beta + 1)}\left[\frac{m_1m_3\beta + 2m_1 - 4{m_3}^2\beta - 5m_3}{{m_3}^2} + \frac{(1 + m_1\beta)(m_1\beta - 4m_3\beta - 3)A_3}{2\sqrt{(m_3 - m_1)(m_3\beta + 1)m_1}}\right]\right\} \tag{A43}$$

$$\left(C_{h_\delta}\right)_0 = -\frac{8m_1(m_1\beta + 3)}{\pi(1 + m_1\beta)^2} \tag{A44}$$

$$\left(C_{L_\delta}\right)_f = \frac{36m_1}{\pi(1 + m_1\beta)} \tag{A45}$$

For configurations having $m_2\beta = -1$ and $m_3\beta = \infty$, the formulas for the characteristics resulting from equations (A1) to (A4) are:

$$C_{L_\delta} = \frac{8m_1}{\pi}\left[\frac{2}{1 + m_1\beta} - 1 - \frac{(1 + m_1\beta)A_4}{\sqrt{m_1\beta}}\right] \tag{A46}$$

$$C_{l_\delta} = \frac{4(1 + m_1\beta)}{3\pi}\left\{\frac{16m_1}{3(1 + m_1\beta)^2} - \frac{1}{\beta}\left[3m_1\beta + 1 + \frac{(1 + m_1\beta)(3m_1\beta - 1)A_4}{2\sqrt{m_1\beta}}\right]\right\} \tag{A47}$$

$$C_{m_\delta} = -\frac{4}{3\pi}\left\{\frac{8(m_1\beta + 3)}{3(1 + m_1\beta)^2} - 2\left[2 + \frac{(1 + m_1\beta)A_4}{\sqrt{m_1\beta}}\right]\right\} \tag{A48}$$

$$\left(C_{h_\delta}\right)_0 = -\frac{8m_1(m_1\beta + 3)}{\pi(1 + m_1\beta)^2} \tag{A49}$$

$$\left(C_{L_\delta}\right)_f = \frac{36m_1}{\pi(1 + m_1\beta)} \tag{A50}$$

## APPENDIX B

## GENERALIZED AND SPECIFIC FORMULAS FOR $C_{L_\delta}$, $C_{l_\delta}$, $C_{m_\delta}$, AND $C_{h_\delta}$ FOR SUPERSONIC LEADING EDGES

The generalized formulas for the derivation of the characteristics $C_{L_\delta}$, $C_{l_\delta}$, $C_{m_\delta}$, and $C_{h_\delta}$ for supersonic leading edges are:

$$C_{L_\delta} = \frac{4\beta(m_2 - m_1)}{\pi m_2\sqrt{m_1^2\beta^2 - 1}}\left[m_2^2 \int_{t_a=0}^{t_a=m_1\beta} \cos^{-1}\left(\frac{1 - m_1\beta t_a}{m_1\beta - t_a}\right)\frac{dt_a}{(m_2\beta - t_a)^2} + m_3^2 \int_{t_a=-1}^{t_a=0} \cos^{-1}\left(\frac{1 - m_1\beta t_a}{m_1\beta - t_a}\right)\frac{dt_a}{(m_3\beta - t_a)^2}\right] \tag{B1}$$

$$C_{l_\delta} = \frac{8\beta(m_2 - m_1)^2}{3\pi m_1 m_2^2\sqrt{m_1^2\beta^2 - 1}}\left[m_2^3 \int_0^{m_1\beta} \cos^{-1}\left(\frac{1 - m_1\beta t_a}{m_1\beta - t_a}\right)\frac{t_a\,dt_a}{(m_2\beta - t_a)^3} + m_3^3 \int_{-1}^{0} \cos^{-1}\left(\frac{1 - m_1\beta t_a}{m_1\beta - t_a}\right)\frac{t_a\,dt_a}{(m_3\beta - t_a)^3}\right] \tag{B2}$$

$$C_{m_\delta} = -\frac{8\beta^2(m_2 - m_1)}{3\pi m_2\sqrt{m_1^2\beta^2 - 1}}\left[m_2^3 \int_0^{m_1\beta} \cos^{-1}\left(\frac{1 - m_1\beta t_a}{m_1\beta - t_a}\right)\frac{dt_a}{(m_2\beta - t_a)^3} + m_3^3 \int_{-1}^{0} \cos^{-1}\left(\frac{1 - m_1\beta t_a}{m_1\beta - t_a}\right)\frac{dt_a}{(m_3\beta - t_a)^3}\right] \tag{B3}$$

$$C_{h_\delta} = -\frac{3m_2\beta\left(m_2 - m_1\right)}{\pi\sqrt{m_1{}^2\beta^2 - 1}}\left[2m_2\beta\int_0^{m_1\beta}\cos^{-1}\left(\frac{1 - m_1\beta t_a}{m_1\beta - t_a}\right)\frac{dt_a}{\left(m_2\beta - t_a\right)^3} - \right.$$

$$\left. 3x_h'\int_0^{m_1\beta}\cos^{-1}\left(\frac{1 - m_1\beta t_a}{m_1\beta - t_a}\right)\frac{dt_a}{\left(m_2\beta - t_a\right)^2}\right] \tag{B4}$$

For the basic configuration $\left(m_2 = m_3 = m_{TE}\right)$, the formulas for the characteristics resulting from equations (B1) to (B4) are:

$$C_{L_\delta} = \frac{4m_{TE}}{\sqrt{m_{TE}{}^2\beta^2 - 1}} \tag{B5}$$

$$C_{l_\delta} = \frac{4m_{TE}\left(m_1 m_{TE}{}^2\beta^2 + m_{TE} - 2m_1\right)}{3m_1\left(m_{TE}{}^2\beta^2 - 1\right)^{3/2}} \tag{B6}$$

$$C_{m_\delta} = \frac{4m_{TE}{}^2\left(m_1 m_{TE}\beta^2 - 2m_{TE}{}^2\beta^2 + 1\right)}{3\left(m_1 - m_{TE}\right)\left(m_{TE}{}^2\beta^2 - 1\right)^{3/2}} \tag{B7}$$

$$C_{h_\delta} = \left(C_{h_\delta}\right)_0 + x_h'\left(C_{L_\delta}\right)_f \tag{B8}$$

$$\left(C_{h_\delta}\right)_0 = -\frac{3m_{TE}^2}{\pi\left(m_1 - m_{TE}\right)\sqrt{m_1^2\beta^2 - 1}}\left[\frac{m_1\left(m_1 - 2m_{TE}\right)\cos^{-1}\left(\frac{1}{m_1\beta}\right)}{m_{TE}^2} + \right.$$

$$\frac{\left(m_1 - m_{TE}\right)\sqrt{m_1^2\beta^2 - 1}}{m_{TE}\left(m_{TE}^2\beta^2 - 1\right)} -$$

$$\left.\frac{2m_{TE}^2\beta^2 - m_1 m_{TE}\beta^2 - 1}{m_{TE}^2\beta^2 - 1}\sqrt{\frac{m_1^2\beta^2 - 1}{m_{TE}^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_{TE}\beta}\right)\right] \tag{B9}$$

$$\left(C_{L_\delta}\right)_f = \frac{9m_{TE}}{\pi\sqrt{m_1^2\beta^2 - 1}}\left[\frac{m_1\cos^{-1}\left(\frac{1}{m_1\beta}\right)}{m_{TE}} + \sqrt{\frac{m_1^2\beta^2 - 1}{m_{TE}^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_{TE}\beta}\right)\right] \tag{B10}$$

For configurations having $1 < \left|m_2\beta\right| < \infty$ and $1 < \left|m_3\beta\right| < \infty$, the formulas for the characteristics resulting from equations (B1) to (B4) are:

$$C_{L_\delta} = \frac{4\left(m_1 - m_2\right)}{\pi m_2\sqrt{m_1^2\beta^2 - 1}}\left\{\frac{m_2^2}{m_1 - m_2}\left[\frac{m_1}{m_2}\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \sqrt{\frac{m_1^2\beta^2 - 1}{m_2^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right)\right] - \right.$$

$$\left.\frac{m_3^2}{m_1 - m_3}\left[\frac{m_1}{m_3}\cos^{-1}\left(\frac{1}{m_1\beta}\right) - \sqrt{\frac{m_1^2\beta^2 - 1}{m_3^2\beta^2 - 1}}\cos^{-1}\left(\frac{1}{m_3\beta}\right)\right]\right\} \tag{B11}$$

$$C_{l_\delta} = \frac{4(m_1 - m_2)^2}{3\pi m_1 m_2^2\sqrt{m_1^2\beta^2 - 1}}\left\{\frac{m_2^3}{m_1 - m_2}\left[\frac{m_1^2\cos^{-1}\left(\frac{1}{m_1\beta}\right)}{m_2(m_1 - m_2)} - \frac{\sqrt{m_1^2\beta^2 - 1}}{m_2^2\beta^2 - 1} + \right.\right.$$

$$\left.\frac{m_1 m_2^2\beta^2 + m_2 - 2m_1}{(m_1 - m_2)(m_2^2\beta^2 - 1)}\sqrt{\frac{m_1^2\beta^2 - 1}{m_2^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right)\right] -$$

$$\frac{m_3^3}{m_1 - m_3}\left[\frac{m_1^2\cos^{-1}\left(\frac{1}{m_1\beta}\right)}{m_3(m_1 - m_3)} - \frac{\sqrt{m_1^2\beta^2 - 1}}{m_3^2\beta^2 - 1} - \right.$$

$$\left.\left.\frac{m_1 m_3^2\beta^2 + m_3 - 2m_1}{(m_1 - m_3)(m_3^2\beta^2 - 1)}\sqrt{\frac{m_1^2\beta^2 - 1}{m_3^2\beta^2 - 1}}\cos^{-1}\left(\frac{1}{m_3\beta}\right)\right]\right\} \tag{B12}$$

$$C_{m_\delta} = -\frac{4(m_1 - m_2)}{3\pi m_2\sqrt{m_1^2\beta^2 - 1}}\left\{\frac{m_2^3}{m_1 - m_2}\left[\frac{m_1(m_1 - 2m_2)\cos^{-1}\left(\frac{1}{m_1\beta}\right)}{m_2^2(m_1 - m_2)} + \frac{\sqrt{m_1^2\beta^2 - 1}}{m_2(m_2^2\beta^2 - 1)} + \right.\right.$$

$$\left.\frac{m_1 m_2\beta^2 - 1 - 2m_2^2\beta^2}{(m_1 - m_2)(m_2^2\beta^2 - 1)}\sqrt{\frac{m_1^2\beta^2 - 1}{m_2^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right)\right] -$$

$$\frac{m_3^3}{m_1 - m_3}\left[\frac{m_1(m_1 - 2m_3)\cos^{-1}\left(\frac{1}{m_1\beta}\right)}{m_3^2(m_1 - m_3)} + \frac{\sqrt{m_1^2\beta^2 - 1}}{m_3(m_3^2\beta^2 - 1)} - \right.$$

$$\left.\left.\frac{m_1 m_3\beta^2 + 1 - 2m_3^2\beta^2}{(m_1 - m_3)(m_3^2\beta^2 - 1)}\sqrt{\frac{m_1^2\beta^2 - 1}{m_3^2\beta^2 - 1}}\cos^{-1}\left(\frac{1}{m_3\beta}\right)\right]\right\} \tag{B13}$$

$$\left(C_{h_\delta}\right)_0 = -\frac{3m_2^2}{\pi\left(m_1 - m_2\right)\sqrt{m_1^2\beta^2 - 1}}\left[\frac{m_1\left(m_1 - 2m_2\right)\cos^{-1}\left(\frac{1}{m_1\beta}\right)}{m_2^2} + \right.$$

$$\left.\frac{\left(m_1 - m_2\right)\sqrt{m_1^2\beta^2 - 1}}{m_2\left(m_2^2\beta^2 - 1\right)} + \frac{m_1m_2\beta^2 + 1 - 2m_2^2\beta^2}{m_2^2\beta^2 - 1}\sqrt{\frac{m_1^2\beta^2 - 1}{m_2^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right)\right] \tag{B14}$$

$$\left(C_{L_\delta}\right)_f = \frac{9m_2}{\pi\sqrt{m_1^2\beta^2 - 1}}\left[\frac{m_1}{m_2}\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \sqrt{\frac{m_1^2\beta^2 - 1}{m_2^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right)\right] \tag{B15}$$

For configurations having $1 < \left|m_2\beta\right| < \infty$ and $m_3\beta = \infty$, the formulas for the characteristics resulting from equations (B1) to (B4) are:

$$C_{L_\delta} = \frac{4\left(m_1 - m_2\right)}{\pi m_2\sqrt{m_1^2\beta^2 - 1}}\left\{\frac{m_2^2}{m_1 - m_2}\left[\frac{m_1}{m_2}\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \sqrt{\frac{m_1^2\beta^2 - 1}{m_2^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right)\right] + \right.$$

$$\left. m_1\cos^{-1}\left(\frac{1}{m_1\beta}\right) - \frac{\pi}{2\beta}\sqrt{m_1^2\beta^2 - 1}\right\} \tag{B16}$$

$$C_{l_\delta} = \frac{4\left(m_1 - m_2\right)^2}{3\pi m_1m_2^2\sqrt{m_1^2\beta^2 - 1}}\left\{\frac{m_2^3}{m_1 - m_2}\left[\frac{m_1^2\cos^{-1}\left(\frac{1}{m_1\beta}\right)}{m_2\left(m_1 - m_2\right)} - \frac{\sqrt{m_1^2\beta^2 - 1}}{m_2^2\beta^2 - 1} + \right.\right.$$

$$\left.\frac{m_1m_2^2\beta^2 + m_2 - 2m_1}{\left(m_1 - m_2\right)\left(m_2^2\beta^2 - 1\right)}\sqrt{\frac{m_1^2\beta^2 - 1}{m_2^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right)\right] -$$

$$\left. m_1^2\cos^{-1}\left(\frac{1}{m_1\beta}\right) - \frac{1}{\beta^2}\sqrt{m_1^2\beta^2 - 1} + \frac{m_1\pi}{2\beta}\sqrt{m_1^2\beta^2 - 1}\right\} \tag{B17}$$

$$C_{m_\delta} = -\frac{4(m_1 - m_2)}{3\pi m_2\sqrt{m_1^2\beta^2 - 1}}\left\{\frac{m_2^3}{m_1 - m_2}\left[\frac{m_1(m_1 - 2m_2)}{m_2^2(m_1 - m_2)}\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{\sqrt{m_1^2\beta^2 - 1}}{m_2(m_2^2\beta^2 - 1)} + \right.\right.$$

$$\left.\frac{m_1m_2\beta^2 + 1 - 2m_2^2\beta^2}{(m_1 - m_2)(m_2^2\beta^2 - 1)}\sqrt{\frac{m_1^2\beta^2 - 1}{m_2^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right)\right] +$$

$$\left. 2m_1\cos^{-1}\left(\frac{1}{m_1\beta}\right) - \frac{\pi}{\beta}\sqrt{m_1^2\beta^2 - 1}\right\} \tag{B18}$$

$$\left(C_{h_\delta}\right)_0 = -\frac{3m_2^2}{\pi(m_1 - m_2)\sqrt{m_1^2\beta^2 - 1}}\left[\frac{m_1(m_1 - 2m_2)\cos^{-1}\left(\frac{1}{m_1\beta}\right)}{m_2^2} + \right.$$

$$\left.\frac{(m_1 - m_2)\sqrt{m_1^2\beta^2 - 1}}{m_2(m_2^2\beta^2 - 1)} + \frac{m_1m_2\beta^2 + 1 - 2m_2^2\beta^2}{m_2^2\beta^2 - 1}\sqrt{\frac{m_1^2\beta^2 - 1}{m_2^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right)\right] \tag{B19}$$

$$\left(C_{L_\delta}\right)_f = \frac{9m_2}{\pi\sqrt{m_1^2\beta^2 - 1}}\left[\frac{m_1}{m_2}\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \sqrt{\frac{m_1^2\beta^2 - 1}{m_2^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right)\right] \tag{B20}$$

For configurations having $1 < |m_2\beta| < \infty$ and $m_3\beta = 1$, the formulas for the characteristics resulting from equations (B1) to (B4) are:

$$C_{L_\delta} = \frac{4(m_1 - m_2)}{\pi m_2\sqrt{m_1^2\beta^2 - 1}}\left\{\frac{m_2^2}{m_1 - m_2}\left[\frac{m_1}{m_2}\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \sqrt{\frac{m_1^2\beta^2 - 1}{m_2^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right)\right] - \right.$$

$$\left.\frac{1}{m_1\beta - 1}\left[m_1\cos^{-1}\left(\frac{1}{m_1\beta}\right) - \frac{1}{\beta}\sqrt{m_1^2\beta^2 - 1}\right]\right\} \tag{B21}$$

$$
C_{l_\delta} = \frac{4\left(m_1 - m_2\right)^2}{3\pi m_1 m_2{}^2\sqrt{m_1{}^2\beta^2 - 1}}\left\{\frac{m_2{}^3}{m_1 - m_2}\left[\frac{m_1{}^2\cos^{-1}\left(\frac{1}{m_1\beta}\right)}{m_2\left(m_1 - m_2\right)} - \frac{\sqrt{m_1{}^2\beta^2 - 1}}{m_2{}^2\beta^2 - 1} + \right.\right.
$$

$$
\left.\frac{m_1 m_2{}^2\beta^2 + m_2 - 2m_1}{\left(m_1 - m_2\right)\left(m_2{}^2\beta^2 - 1\right)}\sqrt{\frac{m_1{}^2\beta^2 - 1}{m_2{}^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right)\right] -
$$

$$
\left.\left(\frac{1}{m_1\beta - 1}\right)^2\left[m_1{}^2\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{\left(1 - 4m_1\beta\right)\sqrt{m_1{}^2\beta^2 - 1}}{3\beta}\right]\right\} \tag{B22}
$$

$$
C_{m_\delta} = -\frac{4\left(m_1 - m_2\right)}{3\pi m_2\sqrt{m_1{}^2\beta^2 - 1}}\left\{\frac{m_2{}^3}{m_1 - m_2}\left[\frac{m_1\left(2m_2 - m_1\right)}{m_2{}^2\left(m_1 - m_2\right)}\cos^{-1}\left(\frac{1}{m_1\beta}\right) - \right.\right.
$$

$$
\left.\frac{\sqrt{m_1{}^2\beta^2 - 1}}{m_2\left(m_2{}^2\beta^2 - 1\right)} - \frac{m_1 m_2\beta^2 + 1 - 2m_2{}^2\beta^2}{\left(m_1 - m_2\right)\left(m_2{}^2\beta^2 - 1\right)}\sqrt{\frac{m_1{}^2\beta^2 - 1}{m_2{}^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right)\right] +
$$

$$
\left.\left(\frac{1}{m_1\beta - 1}\right)^2\left[m_1\left(2 - m_1\beta\right)\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{\left(2m_1\beta - 5\right)\sqrt{m_1{}^2\beta^2 - 1}}{3\beta}\right]\right\} \tag{B23}
$$

$$
\left(C_{h_\delta}\right)_0 = -\frac{3m_2{}^2}{\pi\left(m_1 - m_2\right)\sqrt{m_1{}^2\beta^2 - 1}}\left[\frac{m_1\left(m_1 - 2m_2\right)\cos^{-1}\left(\frac{1}{m_1\beta}\right)}{m_2{}^2} + \right.
$$

$$
\left.\frac{\left(m_1 - m_2\right)\sqrt{m_1{}^2\beta^2 - 1}}{m_2\left(m_2{}^2\beta^2 - 1\right)} + \frac{m_1 m_2\beta^2 + 1 - 2m_2{}^2\beta^2}{m_2{}^2\beta^2 - 1}\sqrt{\frac{m_1{}^2\beta^2 - 1}{m_2{}^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right)\right] \tag{B24}
$$

$$\left(C_{L_\delta}\right)_f = \frac{9m_2}{\pi\sqrt{m_1^2\beta^2 - 1}}\left[\frac{m_1}{m_2}\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \sqrt{\frac{m_1^2\beta^2 - 1}{m_2^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_1\beta}\right)\right] \tag{B25}$$

For configurations having $m_2\beta = \infty$ and $1 < |m_3\beta| < \infty$, the formulas for the characteristics resulting from equations (B1) to (B4) are:

$$C_{L_\delta} = \frac{4}{\pi\sqrt{m_1^2\beta^2 - 1}}\left\{m_1\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{\pi}{2\beta}\sqrt{m_1^2\beta^2 - 1} + \right.$$

$$\left.\frac{m_3^2}{m_1 - m_3}\left[\frac{m_1}{m_3}\cos^{-1}\left(\frac{1}{m_1\beta}\right) - \sqrt{\frac{m_1^2\beta^2 - 1}{m_3^2\beta^2 - 1}}\cos^{-1}\left(\frac{1}{m_3\beta}\right)\right]\right\} \tag{B26}$$

$$C_{l_\delta} = \frac{4}{3\pi m_1\sqrt{m_1^2\beta^2 - 1}}\left\{m_1^2\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{1}{\beta^2}\sqrt{m_1^2\beta^2 - 1} + \frac{m_1\pi}{2\beta}\sqrt{m_1^2\beta^2 - 1} + \right.$$

$$\frac{m_3^3}{m_1 - m_3}\left[\frac{m_1^2\cos^{-1}\left(\frac{1}{m_1\beta}\right)}{m_3(m_1 - m_3)} - \frac{\sqrt{m_1^2\beta^2 - 1}}{m_3^2\beta^2 - 1} - \right.$$

$$\left.\left.\frac{m_1m_3^2\beta^2 + m_3 - 2m_1}{(m_1 - m_3)(m_3^2\beta^2 - 1)}\sqrt{\frac{m_1^2\beta^2 - 1}{m_3^2\beta^2 - 1}}\cos^{-1}\left(\frac{1}{m_3\beta}\right)\right]\right\} \tag{B27}$$

$$C_{m_\delta} = -\frac{4}{3\pi\sqrt{m_1{}^2\beta^2 - 1}}\left\{2m_1 \cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{\pi}{\beta}\sqrt{m_1{}^2\beta^2 - 1} - \right.$$

$$\frac{m_3{}^3}{m_1 - m_3}\left[\frac{m_1(2m_3 - m_1)}{m_3{}^2(m_1 - m_3)} \cos^{-1}\left(\frac{1}{m_1\beta}\right) - \frac{\sqrt{m_1{}^2\beta^2 - 1}}{m_3\left(m_3{}^2\beta^2 - 1\right)} + \right.$$

$$\left.\left.\frac{m_1m_3\beta^2 + 1 - 2m_3{}^2\beta^2}{(m_1 - m_3)\left(m_3{}^2\beta^2 - 1\right)}\sqrt{\frac{m_1{}^2\beta^2 - 1}{m_3{}^2\beta^2 - 1}} \cos^{-1}\left(\frac{1}{m_3\beta}\right)\right]\right\} \tag{B28}$$

$$\left(C_{h_\delta}\right)_0 = -\frac{6}{\pi\sqrt{m_1{}^2\beta^2 - 1}}\left[m_1 \cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{\pi}{2\beta}\sqrt{m_1{}^2\beta^2 - 1}\right] \tag{B29}$$

$$\left(C_{L_\delta}\right)_{fl} = -\frac{3}{2}\left(C_{h_\delta}\right)_0 \tag{B30}$$

For configurations having $m_2\beta = \infty$ and $m_3\beta = 1$, the formulas for the characteristics resulting from equations (B1) to (B4) are:

$$C_{L_\delta} = \frac{4}{\pi\sqrt{m_1{}^2\beta^2 - 1}}\left[\frac{m_1 \cos^{-1}\left(\frac{1}{m_1\beta}\right)}{m_1\beta - 1} + \frac{1}{\beta}\left(\frac{\pi}{2} - \frac{1}{m_1\beta - 1}\right)\sqrt{m_1{}^2\beta^2 - 1}\right] \tag{B31}$$

$$C_{l_\delta} = \frac{4}{3\pi m_1\sqrt{m_1{}^2\beta^2 - 1}}\left\{m_1{}^2\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{1}{\beta^2}\sqrt{m_1{}^2\beta^2 - 1} + \frac{m_1\pi}{2\beta}\sqrt{m_1{}^2\beta^2 - 1} - \right.$$

$$\left.\left(\frac{1}{m_1\beta - 1}\right)^2\left[m_1{}^2\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{(1 - 4m_1\beta)\sqrt{m_1{}^2\beta^2 - 1}}{3\beta}\right]\right\} \tag{B32}$$

$$C_{m_\delta} = -\frac{4}{3\pi\sqrt{m_1^2\beta^2 - 1}}\left\{2\left[m_1 \cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{\pi}{2\beta}\sqrt{m_1^2\beta^2 - 1}\right] - \left(\frac{1}{m_1\beta - 1}\right)^2\left[m_1\left(2 - m_1\beta\right)\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{\left(2m_1\beta - 5\right)\sqrt{m_1^2\beta^2 - 1}}{3\beta}\right]\right\} \tag{B33}$$

$$\left(C_{h_\delta}\right)_0 = -\frac{6}{\pi\sqrt{m_1^2\beta^2 - 1}}\left[m_1 \cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{\pi}{2\beta}\sqrt{m_1^2\beta^2 - 1}\right] \tag{B34}$$

$$\left(C_{L_\delta}\right)_f = -\frac{3}{2}\left(C_{h_\delta}\right)_0 \tag{B35}$$

For configurations having $m_2\beta = \infty$ and $m_3\beta = \infty$, the formulas for the characteristics resulting from equations (B1) to (B4) are:

$$C_{L_\delta} = \frac{4}{\beta} \tag{B36}$$

$$C_{l_\delta} = \frac{4}{3\beta} \tag{B37}$$

$$C_{m_\delta} = -\frac{8}{3\beta} \tag{B38}$$

$$\left(C_{h_\delta}\right)_0 = -\frac{6}{\pi\sqrt{m_1^2\beta^2 - 1}}\left[m_1 \cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{\pi}{2\beta}\sqrt{m_1^2\beta^2 - 1}\right] \tag{B39}$$

$$\left(C_{L_\delta}\right)_f = -\frac{3}{2}\left(C_{h_\delta}\right)_0 \tag{B40}$$

For configurations having $m_2\beta = -1$ and $1 < |m_3\beta| < \infty$, the formulas for the characteristics resulting from equations (B1) to (B4) are:

$$C_{L_\delta} = \frac{4(1 + m_1\beta)}{\pi\sqrt{m_1^2\beta^2 - 1}}\left\{\frac{1}{1 + m_1\beta}\left[m_1 \cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{1}{\beta}\sqrt{m_1^2\beta^2 - 1}\right] + \frac{m_3^2}{m_1 - m_3}\left[\frac{m_1}{m_3}\cos^{-1}\left(\frac{1}{m_1\beta}\right) - \sqrt{\frac{m_1^2\beta^2 - 1}{m_3^2\beta^2 - 1}}\cos^{-1}\left(\frac{1}{m_3\beta}\right)\right]\right\} \tag{B41}$$

$$C_{l_\delta} = \frac{4(1 + m_1\beta)^2}{3\pi m_1\sqrt{m_1^2\beta^2 - 1}}\left\{\left(\frac{1}{1 + m_1\beta}\right)^2\left[m_1^2\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{(1 + 4m_1\beta)\sqrt{m_1^2\beta^2 - 1}}{3\beta^2}\right] - \frac{m_3^3}{m_1 - m_3}\left[\frac{m_1^2\cos^{-1}\left(\frac{1}{m_1\beta}\right)}{m_3(m_1 - m_3)} - \frac{\sqrt{m_1^2\beta^2 - 1}}{m_3^2\beta^2 - 1} + \frac{2m_1 - m_3 - m_1m_3^2\beta^2}{(m_1 - m_3)(m_3^2\beta^2 - 1)}\sqrt{\frac{m_1^2\beta^2 - 1}{m_3^2\beta^2 - 1}}\cos^{-1}\left(\frac{1}{m_3\beta}\right)\right]\right\} \tag{B42}$$

$$C_{m_\delta} = -\frac{4(1 + m_1\beta)}{3\pi\sqrt{m_1^2\beta^2 - 1}}\left\{\left(\frac{1}{1 + m_1\beta}\right)^2\left[m_1(m_1\beta + 2)\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{2m_1\beta + 5}{3\beta}\sqrt{m_1^2\beta^2 - 1}\right] - \frac{m_3^3}{m_1 - m_3}\left[\frac{m_1(2m_3 - m_1)}{m_3^2(m_1 - m_3)}\cos^{-1}\left(\frac{1}{m_1\beta}\right) - \frac{\sqrt{m_1^2\beta^2 - 1}}{m_3(m_3^2\beta^2 - 1)} + \frac{m_1m_3\beta^2 + 1 - 2m_3^2\beta^2}{(m_1 - m_3)(m_3^2\beta^2 - 1)}\sqrt{\frac{m_1^2\beta^2 - 1}{m_3^2\beta^2 - 1}}\cos^{-1}\left(\frac{1}{m_3\beta}\right)\right]\right\} \tag{B43}$$

$$\left(C_{h_\delta}\right)_0 = -\frac{3}{\pi\left(1 + m_1\beta\right)\sqrt{m_1^2\beta^2 - 1}}\left[m_1\left(m_1\beta + 2\right)\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{2m_1\beta + 5}{3\beta}\sqrt{m_1^2\beta^2 - 1}\right] \tag{B44}$$

$$\left(C_{L_\delta}\right)_f = \frac{9}{\pi\sqrt{m_1^2\beta^2 - 1}}\left[m_1 \cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{1}{\beta}\sqrt{m_1^2\beta^2 - 1}\right] \tag{B45}$$

For configurations having $m_2\beta = -1$ and $m_3\beta = 1$, the formulas for the characteristics resulting from equations (B1) to (B4) are:

$$C_{L_\delta} = \frac{8\left(1 + m_1\beta\right)}{\pi\left(m_1^2\beta^2 - 1\right)^{3/2}}\left[m_1^2\beta \cos^{-1}\left(\frac{1}{m_1\beta}\right) - \frac{1}{\beta}\sqrt{m_1^2\beta^2 - 1}\right] \tag{B46}$$

$$C_{l_\delta} = \frac{4\left(1 + m_1\beta\right)^2}{3\pi\sqrt{m_1^2\beta^2 - 1}}\left\{\left(\frac{1}{1 + m_1\beta}\right)^2\left[m_1^2\beta \cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{\left(4m_1\beta + 1\right)\sqrt{m_1^2\beta^2 - 1}}{3\beta}\right] - \right.$$

$$\left.\left(\frac{1}{m_1\beta - 1}\right)^2\left[m_1^2\beta \cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{1 - 4m_1\beta}{3\beta}\sqrt{m_1^2\beta^2 - 1}\right]\right\} \tag{B47}$$

$$C_{m_\delta} = -\frac{4\left(1 + m_1\beta\right)}{3\pi\sqrt{m_1^2\beta^2 - 1}}\left\{\left(\frac{1}{m_1\beta + 1}\right)^2\left[m_1\left(m_1\beta + 2\right)\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \right.\right.$$

$$\left.\frac{2m_1\beta + 5}{3\beta}\sqrt{m_1^2\beta^2 - 1}\right] - \left(\frac{1}{m_1\beta - 1}\right)^2\left[m_1\left(2 - m_1\beta\right)\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \right.$$

$$\left.\left.\frac{\left(2m_1\beta - 5\right)\sqrt{m_1^2\beta^2 - 1}}{3\beta}\right]\right\} \tag{B48}$$

$$\left(C_{h_\delta}\right)_0 = -\frac{3}{\pi\left(1 + m_1\beta\right)\sqrt{m_1{}^2\beta^2 - 1}}\left[m_1\left(m_1\beta + 2\right)\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{2m_1\beta + 5}{3\beta}\sqrt{m_1{}^2\beta^2 - 1}\right] \tag{B49}$$

$$\left(C_{L_\delta}\right)_f = \frac{9}{\pi\sqrt{m_1{}^2\beta^2 - 1}}\left[m_1 \cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{1}{\beta}\sqrt{m_1{}^2\beta^2 - 1}\right] \tag{B50}$$

For configurations having $m_2\beta = -1$ and $m_3\beta = \infty$, the formulas for the characteristics resulting from equations (B1) to (B4) are:

$$C_{L_\delta} = \frac{4\left(1 + m_1\beta\right)}{\pi\sqrt{m_1{}^2\beta^2 - 1}}\left\{\frac{1}{m_1\beta + 1}\left[m_1 \cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{1}{\beta}\sqrt{m_1{}^2\beta^2 - 1}\right] - m_1 \cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{\pi}{2\beta}\sqrt{m_1{}^2\beta^2 - 1}\right\} \tag{B51}$$

$$C_{l_\delta} = \frac{4\left(1 + m_1\beta\right)^2}{3\pi m_1\sqrt{m_1{}^2\beta^2 - 1}}\left\{\left(\frac{1}{m_1\beta + 1}\right)^2\left[m_1{}^2\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{\left(4m_1\beta + 1\right)\sqrt{m_1{}^2\beta^2 - 1}}{3\beta^2}\right] - m_1{}^2\cos^{-1}\left(\frac{1}{m_1\beta}\right) - \frac{1}{\beta^2}\sqrt{m_1{}^2\beta^2 - 1} + \frac{m_1\pi}{2\beta}\sqrt{m_1{}^2\beta^2 - 1}\right\} \tag{B52}$$

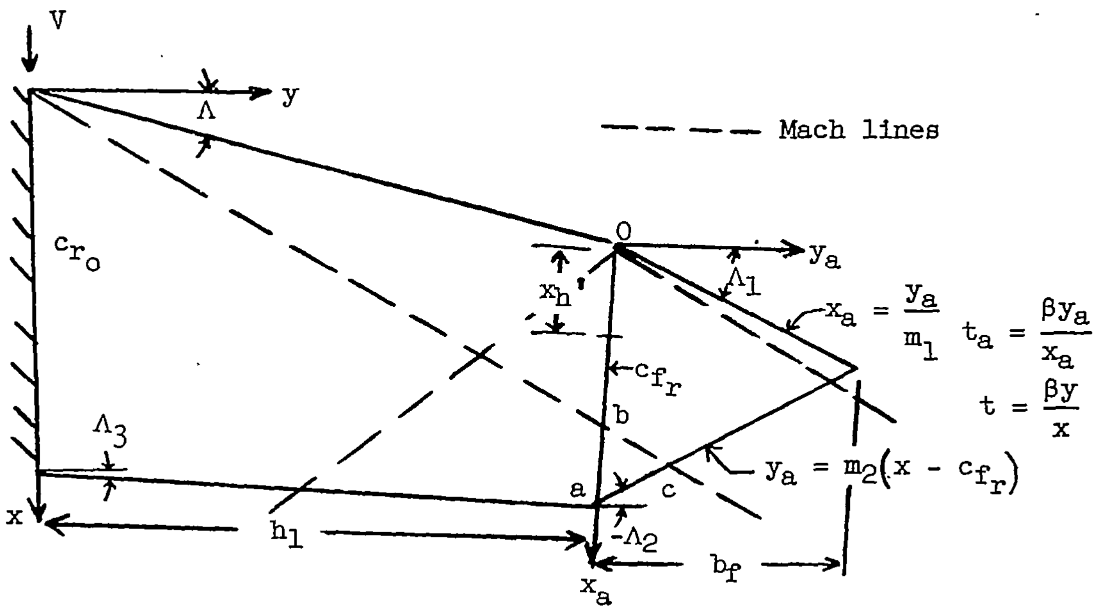
$$C_{m_\delta} = -\frac{4\left(1 + m_1\beta\right)}{3\pi\sqrt{m_1{}^2\beta^2 - 1}}\left\{\left(\frac{1}{m_1\beta + 1}\right)^2\left[m_1\left(m_1\beta + 2\right)\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{\left(2m_1\beta + 5\right)\sqrt{m_1{}^2\beta^2 - 1}}{3\beta}\right] - 2\left[m_1 \cos^{-1}\left(\frac{1}{m_1\beta}\right) - \frac{\pi}{2\beta}\sqrt{m_1{}^2\beta^2 - 1}\right]\right\} \tag{B53}$$

$$\left(C_{h_\delta}\right)_o = -\frac{3}{\pi\left(1 + m_1\beta\right)\sqrt{m_1{}^2\beta^2 - 1}}\left[m_1\left(m_1\beta + 2\right)\cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{2m_1\beta + 5}{3\beta}\sqrt{m_1{}^2\beta^2 - 1}\right] \tag{B54}$$

$$\left(C_{L_\delta}\right)_f = \frac{9}{\pi\sqrt{m_1{}^2\beta^2 - 1}}\left[m_1 \cos^{-1}\left(\frac{1}{m_1\beta}\right) + \frac{1}{\beta}\sqrt{m_1{}^2\beta^2 - 1}\right] \tag{B55}$$

## APPENDIX C

## DERIVATION AND FORMULAS FOR $C_{h_\alpha}$ FOR SUPERSONIC LEADING EDGES




The generalized formula for $C_{h_\alpha}$ for supersonic leading edges is

$$C_{h_\alpha} = \left(C_{h_\alpha}\right)_0 + x_h'\left(C_{L_\alpha}\right)_f \tag{C1}$$

where

$$\left(C_{h_\alpha}\right)_0 = -\left\{\frac{3mm_2^2(m_2 - m_1)}{m_1\sqrt{m^2\beta^2 - 1}}\left[\left(\frac{1}{m_2 - m_1}\right)^2 - \frac{1}{m_2^2}\right] - \frac{6mm_2^2(m_2 - m_1)B_1}{\pi m_1\sqrt{m^2\beta^2 - 1}} + \right.$$

$$\frac{6m_2^2(m_2 - m_1)B_2}{\pi\sqrt{m_1^2\beta^2 - 1}} + \frac{6m_1(m_2 - m_1)\left[m_2 + \frac{h_1}{c_{f_r}}\left(\frac{m_2}{m_1} - 1\right)\right]^3 B_3}{\pi m_1 m_2\sqrt{m^2\beta^2 - 1}} -$$

$$\left.\frac{6m\left(\frac{h_1}{c_{f_r}}\right)^3(m_2 - m_1)B_4}{\pi m_1 m_2\sqrt{m^2\beta^2 - 1}} - \frac{18m(m_2 - m_1)B_5}{m_1 m_2\sqrt{m^2\beta^2 - 1}}\right\} \tag{C2}$$

$$\left(C_{L_\alpha}\right)_f = \frac{9m}{\sqrt{m^2\beta^2 - 1}} - \frac{9mm_2(m_2 - m_1)B_8}{\pi m_1\sqrt{m^2\beta^2 - 1}} + \frac{9m_2(m_2 - m_1)B_9}{\pi\sqrt{m_1^2\beta^2 - 1}} +$$

$$\frac{9m\left[m_2 + \frac{h_1}{c_{f_r}}\left(\frac{m_2}{m} - 1\right)\right]^2(m_2 - m_1)B_6}{\pi m_1 m_2\sqrt{m^2\beta^2 - 1}} - \frac{9m(m_2 - m_1)\left(\frac{h_1}{c_{f_r}}\right)^2 B_7}{\pi m_1 m_2\sqrt{m^2\beta^2 - 1}} -$$

$$\frac{9(m_2 - m_1)\left[m + \left(\frac{h_1}{c_{f_r}}\right)(1 - m\beta)\right]\left[m_1 + \left(\frac{h_1}{c_{f_r}}\right)(1 - m_1\beta)\right]}{mm_1(m_2\beta - 1)\sqrt{m^2\beta^2 - 1}} \tag{C3}$$

and

$$B_1 = \int_0^{m_1\beta} \cos^{-1}\left(\frac{1 - m\beta t_a}{m\beta - t_a}\right)\frac{dt_a}{\left(m_2\beta - t_a\right)^3}$$

$$= \frac{1}{2}\left(\frac{\cos^{-1}\left[\frac{1 - mm_1\beta^2}{\beta\left(m - m_1\right)}\right]}{\left(m_2 - m_1\right)^2} - \frac{\cos^{-1}\left(\frac{1}{m\beta}\right)}{m_2^2} - \frac{\sqrt{m^2\beta^2 - 1}}{m_2\left(m - m_2\right)\left(m_2^2\beta^2 - 1\right)} - \right.$$

$$\frac{mm_2\beta^2 - 2m_2^2\beta^2 + 1}{\left(m - m_2\right)^2\left(m_2^2\beta^2 - 1\right)}\sqrt{\frac{m^2\beta^2 - 1}{m_2^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right) -$$

$$\left.\left(\frac{1}{m - m_2}\right)^2\left\{\cos^{-1}\left[\frac{1 - mm_1\beta^2}{\beta\left(m - m_1\right)}\right] - \cos^{-1}\left(\frac{1}{m\beta}\right)\right\}\right) \tag{C4}$$

$$B_2 = \int_0^{m_1\beta} \cos^{-1}\left(\frac{1 - m_1\beta t_a}{m_1\beta - t_a}\right)\frac{dt_a}{\left(m_2\beta - t_a\right)^3}$$

$$= \frac{1}{2}\left[\frac{m_1\left(2m_2 - m_1\right)}{m_2^2\left(m_2 - m_1\right)^2}\cos^{-1}\left(\frac{1}{m_1\beta}\right) - \frac{\sqrt{m_1^2\beta^2 - 1}}{m_2\left(m_1 - m_2\right)\left(m_2^2\beta^2 - 1\right)} - \right.$$

$$\left.\frac{m_1m_2\beta^2 - 2m_2^2\beta^2 + 1}{\left(m_1 - m_2\right)^2\left(m_2^2\beta^2 - 1\right)}\sqrt{\frac{m_1^2\beta^2 - 1}{m_2^2\beta^2 - 1}}\cos^{-1}\left(-\frac{1}{m_2\beta}\right)\right] \tag{C5}$$

$$B_3 = \int_{\nu}^{1} \left[\cos^{-1}\left(\frac{1 - m\beta t}{m\beta - t}\right) + \cos^{-1}\left(\frac{1 + m\beta t}{m\beta + t}\right)\right] \frac{dt}{\left(m_2\beta - t\right)^3}$$

$$= \frac{1}{2}\left\{\frac{\pi\left(m^2\beta^2 - 2mm_2\beta^2 + 2m_2\beta - 1\right)}{\left(m_2\beta - 1\right)^2\left(m - m_2\right)^2} - \right.$$

$$\frac{1}{\left(m_2 - \nu\right)^2}\left[\cos^{-1} \frac{1 - m\beta^2\nu}{\beta(m - \nu)} + \cos^{-1} \frac{1 + m\beta^2\nu}{\beta(m + \nu)}\right] -$$

$$\frac{2m_2\sqrt{\left(1 - \beta^2\nu^2\right)\left(m^2\beta^2 - 1\right)}}{\left(m - m_2\right)\left(m + m_2\right)\left(m_2 - \nu\right)\left(m_2{}^2\beta^2 - 1\right)} + \frac{\cos^{-1}\left[\frac{1 - m\beta^2\nu}{\beta(m - \nu)}\right]}{\left(m - m_2\right)^2} +$$

$$\frac{\cos^{-1}\left[\frac{1 + m\beta^2\nu}{\beta(m + \nu)}\right]}{\left(m + m_2\right)^2} + \frac{\sqrt{m^2\beta^2 - 1}}{\left(m_2{}^2\beta^2 - 1\right)^{3/2}}\left[\frac{mm_2\beta^2 + 2m_2{}^2\beta^2 - 1}{\left(m + m_2\right)^2} - \right.$$

$$\left.\left.\frac{mm_2\beta^2 - 2m_2{}^2\beta^2 + 1}{\left(m - m_2\right)^2}\right]\cos^{-1}\left[\frac{1 - m_2\beta^2\nu}{\beta\left(\nu - m_2\right)}\right]\right\} \tag{C6}$$

$$\nu = \frac{1}{\frac{1}{m_3} + \frac{c_{r_o}}{h_1}} \tag{C7}$$

$$B_4 = \int_{\nu}^{1} \left[\cos^{-1}\left(\frac{1 - m\beta t}{m\beta - t}\right) + \cos^{-1}\left(\frac{1 + m\beta t}{m\beta + t}\right)\right]\frac{dt}{t^3}$$

$$= \frac{(m - \nu)(m + \nu)}{2m^2\nu^2}\left\{\cos^{-1}\left[\frac{1 - m\beta^2\nu}{\beta(m - \nu)}\right] + \cos^{-1}\left[\frac{1 + m\beta^2\nu}{\beta(m + \nu)}\right]\right\} -$$

$$\frac{\sqrt{m^2\beta^2 - 1}}{m^2}\tanh^{-1}\left(-\frac{1}{\sqrt{1 - \beta^2\nu^2}}\right) + \frac{\pi(1 - m\beta)(1 + m\beta)}{2m^2} \tag{C8}$$

$$B_5 = \frac{\left[m_2 + \frac{h_1}{c_{f_r}}\left(\frac{m_2}{m_1} - 1\right)\right]^3}{6}\left[\left(\frac{\beta}{m_2\beta - 1}\right)^2 - \left(\frac{1}{m_2 - \nu}\right)^2\right] + \frac{1}{6}\left(\frac{h_1}{c_{f_r}}\right)^3\left(\beta^2 - \frac{1}{\nu^2}\right) \tag{C9}$$

$$B_6 = \int_{\nu}^{1} \left[\cos^{-1}\left(\frac{1 - m\beta t}{m\beta - t}\right) + \cos^{-1}\left(\frac{1 + m\beta t}{m\beta + t}\right)\right]\frac{dt}{(m_2\beta - t)^2}$$

$$= \frac{\pi(m\beta - 1)}{(m_2\beta - 1)(m - m_2)} - \frac{2m_2}{(m - m_2)(m + m_2)}\sqrt{\frac{m^2\beta^2 - 1}{m_2^2\beta^2 - 1}}\cos^{-1}\left[\frac{1 - m_2\beta^2\nu}{\beta(\nu - m_2)}\right] -$$

$$\frac{1}{m_2 - \nu}\left\{\cos^{-1}\left[\frac{1 - m\beta^2\nu}{\beta(m - \nu)}\right] + \cos^{-1}\left[\frac{1 + m\beta^2\nu}{\beta(m + \nu)}\right]\right\} - \frac{\cos^{-1}\left[\frac{1 - m\beta^2\nu}{\beta(m - \nu)}\right]}{m - m_2} +$$

$$\cos^{-1}\left[\frac{1 + m\beta^2\nu}{\beta(m + \nu)}\right] \tag{C10}$$

$$B_7 = \int_{\nu}^{1} \left[\cos^{-1}\left(\frac{1 - m\beta t}{m\beta - t}\right) + \cos^{-1}\left(\frac{1 + m\beta t}{m\beta + t}\right)\right]\frac{dt}{t^2}$$

$$= \frac{\pi(1 - m\beta)}{m} + \frac{1}{\nu}\left\{\cos^{-1}\left[\frac{1 - m\beta^2\nu}{\beta(m - \nu)}\right] + \cos^{-1}\left[\frac{1 + m\beta^2\nu}{\beta(m + \nu)}\right]\right\} +$$

$$\frac{1}{m}\left\{\cos^{-1}\left[\frac{1 + m\beta^2\nu}{\beta(m + \nu)}\right] - \cos^{-1}\left[\frac{1 - m\beta^2\nu}{\beta(m - \nu)}\right]\right\} \tag{C11}$$

$$B_8 = \int_0^{m_1\beta} \cos^{-1}\left(\frac{1 - m\beta t_a}{m\beta - t_a}\right)\frac{dt_a}{\left(m_2\beta - t_a\right)^2}$$

$$= \frac{m\cos^{-1}\left(\frac{1}{m\beta}\right)}{m_2\left(m_2 - m_1\right)} + \frac{\left(m - m_1\right)\cos^{-1}\left[\frac{1 - mm_1\beta^2}{\beta\left(m - m_1\right)}\right]}{\left(m - m_2\right)\left(m_2 - m_1\right)} -$$

$$\frac{1}{m - m_2}\sqrt{\frac{m^2\beta^2 - 1}{m_2{}^2\beta^2 - 1}}\left\{\cos^{-1}\left(-\frac{1}{m_2\beta}\right) - \cos^{-1}\left[\frac{1 - m_1m_2\beta^2}{\beta\left(m_1 - m_2\right)}\right]\right\} \tag{C12}$$

$$B_9 = \int_0^{m_1\beta} \cos^{-1}\left(\frac{1 - m_1\beta t_a}{m_1\beta - t_a}\right)\frac{dt_a}{\left(m_2\beta - t_a\right)^2}$$

$$= \frac{m_1\cos^{-1}\left(\frac{1}{m_1\beta}\right)}{m_2\left(m_2 - m_1\right)} - \sqrt{\frac{m_1{}^2\beta^2 - 1}{m_2{}^2\beta^2 - 1}}\,\frac{\cos^{-1}\left(-\frac{1}{m_2\beta}\right)}{m_1 - m_2} \tag{C13}$$

When the Mach waves from the wing root-chord apex do not intersect the wing tip chord, $m_3\beta \leqq \frac{1}{1 - \frac{c_{r_o}}{\beta h_1}}$, and equations (C2) and (C3) simplify to

$$\left(C_{h_\alpha}\right)_o = -\left\{\frac{3mm_2^{\,2}\left(m_2 - m_1\right)}{m_1\sqrt{m_1^{\,2}\beta^2 - 1}}\left[\left(\frac{1}{m_2 - m_1}\right)^2 - \left(\frac{1}{m_2}\right)^2\right] - \frac{6mm_2^{\,2}\left(m_2 - m_1\right)B_1}{\pi m_1\sqrt{m^2\beta^2 - 1}} + \frac{6m_2^{\,2}\left(m_2 - m_1\right)B_2}{\pi\sqrt{m_1^{\,2}\beta^2 - 1}}\right\} \tag{C14}$$

$$\left(C_{L_\alpha}\right)_f = \frac{9m}{\sqrt{m^2\beta^2 - 1}} - \frac{9mm_2\left(m_2 - m_1\right)B_8}{\pi m_1\sqrt{m^2\beta^2 - 1}} + \frac{9m_2\left(m_2 - m_1\right)B_9}{\pi\sqrt{m_1^{\,2}\beta^2 - 1}} \tag{C15}$$

If, in addition to this condition, $\Lambda = \Lambda_1$ (that is, $m = m_1$), equations (C14) and (C15) are reduced to

$$\left(C_{h_\alpha}\right)_o = -\frac{3m_2^{\,2}\left(m_2 - m\right)}{\sqrt{m^2\beta^2 - 1}}\left[\left(\frac{1}{m_2 - m}\right)^2 - \left(\frac{1}{m_2}\right)^2\right] \tag{C16}$$

$$\left(C_{L_\alpha}\right)_f = \frac{9m}{\sqrt{m^2\beta^2 - 1}} \tag{C17}$$

When the Mach waves from the wing root-chord apex intersect the tip chord, $m_3\beta > \dfrac{1}{1 - \dfrac{c_{r_0}}{\beta h_1}}$, and $\Lambda = \Lambda_1$, equations (C2) and (C3) become

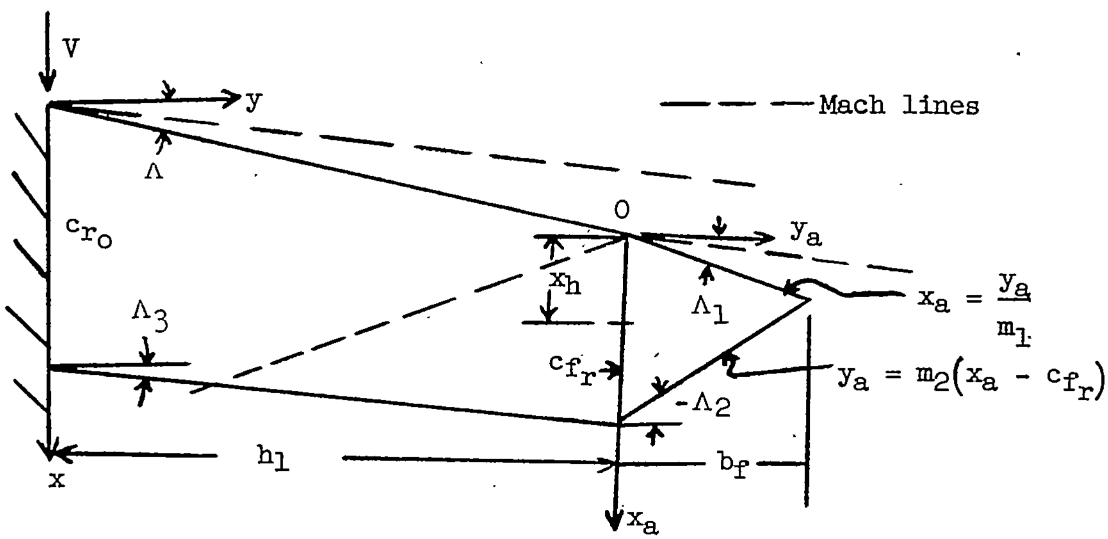
$$\left(C_{h_\alpha}\right)_0 = -\left\{\frac{3m_2^{\,2}\left(m_2 - m_1\right)}{\sqrt{m^2\beta^2 - 1}}\left[\left(\frac{1}{m_2 - m}\right)^2 - \left(\frac{1}{m_2}\right)^2\right] + \frac{6\left(m_2 - m\right)\left[m_2 + \frac{h_1}{c_{f_r}}\left(\frac{m_2}{m} - 1\right)\right]^3 B_3}{\pi m_2\sqrt{m^2\beta^2 - 1}} - \frac{6\left(m_2 - m\right)\left(\frac{h_1}{c_{f_r}}\right)^3 B_4}{\pi m_2\sqrt{m^2\beta^2 - 1}} - \frac{18\left(m_2 - m\right)B_5}{m_2\sqrt{m^2\beta^2 - 1}}\right\} \tag{C18}$$

$$\left(C_{L_\alpha}\right)_f = \frac{9m}{\sqrt{m^2\beta^2 - 1}} + \frac{9\left(m_2 - m\right)}{\pi m_2\sqrt{m^2\beta^2 - 1}}\left[m_2 + \frac{h_1}{c_{f_r}}\left(\frac{m_2}{m} - 1\right)\right]^2 B_6 - \frac{9\left(m_2 - m\right)\left(\frac{h_1}{c_{f_r}}\right)^2 B_7}{\pi m_2\sqrt{m^2\beta^2 - 1}} - \frac{9\left(m_2 - m\right)\left[m + \frac{h_1}{c_{f_r}}\left(1 - m\beta\right)\right]^2}{m^2\left(m_2\beta - 1\right)\sqrt{m^2\beta^2 - 1}} \tag{C19}$$

APPENDIX D

DERIVATION AND FORMULAS FOR $C_{h_\alpha}$ FOR SUBSONIC LEADING EDGES AND

ILLUSTRATIVE CALCULATIONS FOR CONFIGURATION HAVING $\Lambda = \Lambda_1$




Formulas for $C_{h_\alpha}$ for Subsonic Leading Edges

The formula for $C_{h_\alpha}$ for subsonic leading edges is

$$C_{h_\alpha} = \left(C_{h_\alpha}\right)_O + x_h'\left(C_{L_\alpha}\right)_f \tag{D1}$$

where

$$\left(C_{h_\alpha}\right)_0 = \frac{-9\left(m_2 - m_1\right)}{c_{f_r}{}^3 m_1 m_2 E\left(\sqrt{1 - m^2\beta^2}\right)}\sqrt{\frac{1 + m\beta}{1 + m_1\beta}}\int_0^{b_f}\int_{y_a/m_1}^{\frac{y_a}{m_2} + c_{f_r}} x_a\left(m\sqrt{\frac{m_1 x_a - y_a}{m x_a + y_a + 2h_1}} + \right.$$

$$\left. m_1\sqrt{\frac{m x_a + y_a + 2h_1}{m_1 x_a - y_a}}\right)dx_a\, dy_a \tag{D2}$$

$$\left(C_{L_\alpha}\right)_f = \frac{9\left(m_2 - m_1\right)}{c_{f_r}{}^2 m_1 m_2 E\left(\sqrt{1 - m^2\beta^2}\right)}\sqrt{\frac{1 + m\beta}{1 + m_1\beta}}\int_0^{b_f}\int_{y_a/m_1}^{\frac{y_a}{m_2} + c_{f_r}}\left(m\sqrt{\frac{m_1 x_a - y_a}{m x_a + y_a + 2h_1}} + \right.$$

$$\left. m_1\sqrt{\frac{m x_a + y_a + 2h_1}{m_1 x_a - y_a}}\right)dx_a\, dy_a \tag{D3}$$

Solutions to equations (D2) and (D3) are

$$\left(C_{h_\alpha}\right)_0 = -\frac{9\left(m_2 - m_1\right)}{m_1 m_2 E\left(\sqrt{1 - m^2\beta^2}\right)}\sqrt{\frac{1 + m\beta}{1 + m_1\beta}}\left(\frac{1}{12}\left[\frac{1}{2}\left(\frac{1}{m} - \frac{1}{m_1}\right) + \frac{1}{m_2}\right]\left\{\left(A_f{}^2 - \frac{k_1 A_f}{2R} - \right.\right.\right.$$

$$\left.\frac{k_1{}^2}{2R^2} + \frac{k_2{}^2}{R^2}\right)\sqrt{\left[m_1 - \left(1 - \frac{m_1}{m_2}\right)\frac{A_f}{2}\right]\left[\frac{2h_1}{c_{f_r}} + m + \left(1 + \frac{m}{m_2}\right)\frac{A_f}{2}\right]} + \left(\frac{k_1{}^2}{2R^2} - \right.$$

$$\left.\left.\frac{k_2{}^2}{R^2}\right)\sqrt{m_1\left(m + \frac{2h_1}{c_{f_r}}\right)}\right\} + \frac{1}{4}\left(1 - \frac{h_1}{m c_{f_r}}\right)\left\{\left(A_f - \right.\right.$$

$$\left.\frac{k_1}{R}\right)\sqrt{\left[m_1 - \left(1 - \frac{m_1}{m_2}\right)\frac{A_f}{2}\right]\left[\frac{2h_1}{c_{f_r}} + m + \left(1 + \frac{m}{m_2}\right)\frac{A_f}{2}\right]} + \frac{k_1}{R}\sqrt{m_1\left(m + \frac{2h_1}{c_{f_r}}\right)}\right\} +$$

(Equation continued on next page)

$$\frac{k_2^{\,2}}{16R^{5/2}}\left\{k_1\left[\frac{1}{2}\left(\frac{1}{m} - \frac{1}{m_1}\right) + \frac{1}{m_2}\right] + 2R\left(1 - \frac{h_1}{mc_{f_r}}\right)\right\}\left[\sin^{-1}\left(\frac{k_1 - RA_f}{k_2}\right) - \right.$$

$$\left.\sin^{-1}\left(\frac{k_1}{k_2}\right)\right] +$$

$$\frac{\left[\left(m + m_1\right)\frac{A_f}{2} + 2m_1\frac{h_1}{c_{f_r}}\right]^3}{6\left(m + m_1\right)\left(mm_1\right)^{3/2}}\tanh^{-1}\left\{\frac{\sqrt{m\left[m_1 - \left(1 - \frac{m_1}{m_2}\right)\frac{A_f}{2}\right]}}{\sqrt{m_1\left[m + \frac{2h_1}{c_{f_r}} + \left(1 + \frac{m}{m_2}\right)\frac{A_f}{2}\right]}}\right\} -$$

$$\frac{4m_1^{\,3}\left(\frac{h_1}{c_{f_r}}\right)^3}{3\left(m + m_1\right)\left(mm_1\right)^{3/2}}\tanh^{-1}\sqrt{\frac{m}{m + \frac{2h_1}{c_{f_r}}}} -$$

$$\frac{k_2\gamma_1 i}{48mm_1R^{5/2}\left(m + m_1\right)}\left\{\tanh^{-1}\left[\frac{\sqrt{\frac{RA_f}{2} - m_1\left(1 + \frac{m}{m_2}\right)}}{\sqrt{\frac{RA_f}{2} + \left(1 - \frac{m_1}{m_2}\right)\left(m + \frac{2h_1}{c_{f_r}}\right)}}\right] - \right.$$

$$\left.\tanh^{-1}\left[\frac{\sqrt{m_1\left(1 + \frac{m}{m_2}\right)}}{\sqrt{\left(m + \frac{2h_1}{c_{f_r}}\right)\left(\frac{m_1}{m_2} - 1\right)}}\right]\right\} -$$

$$\frac{k_2\gamma_2}{48mm_1R^2}\left\{\sqrt{\left[m_1 - \left(1 - \frac{m_1}{m_2}\right)\frac{A_f}{2}\right]\left[m + \frac{2h_1}{c_{f_r}} + \left(1 + \frac{m}{m_2}\right)\frac{A_f}{2}\right]} - \right.$$

$$\left.\sqrt{m_1\left(m + \frac{2h_1}{c_{f_r}}\right)}\right\} - \frac{k_2\left(m + m_1\right)A_f}{48mm_1R}\sqrt{\left[m_1 - \left(1 - \frac{m_1}{m_2}\right)\frac{A_f}{2}\right]\left[m + \frac{2h_1}{c_{f_r}} + \left(1 + \frac{m}{m_2}\right)\frac{A_f}{2}\right]}\Bigg) \tag{D4}$$

where

$$h_1 = \frac{b}{2} - b_f \tag{D5}$$

$$\left(C_{L_\alpha}\right)_f = \frac{9\left(m_2 - m_1\right)}{2m_1m_2E\left(\sqrt{1 - m^2\beta^2}\right)}\sqrt{\frac{1 + m\beta}{1 + m_1\beta}}\left(\left\{\left(A_f - \right.\right.\right.$$

$$\left.\frac{k_1}{R}\right)\sqrt{\left[m_1 - \left(1 - \frac{m_1}{m_2}\right)\frac{A_f}{2}\right]\left[m + \frac{2h_1}{c_{f_r}} + \left(1 + \frac{m}{m_2}\right)\frac{A_f}{2}\right]} +$$

$$\left.\left.\frac{k_1}{R}\sqrt{m_1\left(m + \frac{2h_1}{c_{f_r}}\right)}\right\} + \frac{k_2^{\,2}}{2R^{3/2}}\left[\sin^{-1}\left(\frac{k_1 - RA_f}{k_2}\right) - \sin^{-1}\left(\frac{k_1}{k_2}\right)\right]\right) \tag{D6}$$

where

$$k_1 = m_1\left(\frac{m}{m_2} + 1\right) + \left(\frac{m_1}{m_2} - 1\right)\left(m + \frac{2h_1}{c_{f_r}}\right) \tag{D7}$$

$$k_2 = m_1\left(\frac{m}{m_2} + 1\right) - \left(\frac{m_1}{m_2} - 1\right)\left(m + \frac{2h_1}{c_{f_r}}\right) \tag{D8}$$

$$R = \left(1 - \frac{m_1}{m_2}\right)\left(1 + \frac{m}{m_2}\right) \tag{D9}$$

$$\gamma_1 = 3(m + m_1)^4 - 8mm_1R(m + m_1)^2 + 32Rmm_1 \frac{h_1}{c_{f_r}}\left(\frac{m_1}{m_2} - 1\right)(m + m_1) +$$

$$12\left(\frac{h_1}{c_{f_r}}\right)^2\left(\frac{m_1}{m_2} - 1\right)^2 (m + m_1)^2 - 32mm_1R\left(\frac{h_1}{c_{f_r}}\right)^2\left(\frac{m_1}{m_2} - 1\right)^2 -$$

$$12 \frac{h_1}{c_{f_r}}\left(\frac{m_1}{m_2} - 1\right)(m + m_1)^3 \tag{D10}$$

$$\gamma_2 = 10m_1R \frac{h_1}{c_{f_r}} - 6m_1 \frac{h_1}{c_{f_r}}\left(\frac{m_1}{m_2} - 1\right)^2 - 3(m + m_1)\left(m - m_1 - \frac{2mm_1}{m_2}\right) \tag{D11}$$

## Illustrative Example

It is desired to calculate $C_{h_\alpha}$ and the travel of the hinge line for the following configuration:

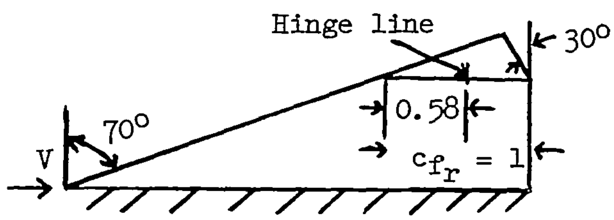
$M = 2$ $\quad \Lambda = \Lambda_1 = 70^\circ$

$\Lambda_2 = -30^\circ$ $\quad m_1 = 0.364$

$m_2 = 1.732$ $\quad \Lambda_3 = 0^\circ$

$\frac{h_1}{c_{f_r}} = 0.6986$ $\quad \frac{A_f}{2} = \frac{b_f}{c_{f_r}} = 0.6062$

$x_h' = 0.582$ $\quad E\left(\sqrt{1 - m^2\beta^2}\right) = 1.297$



From equations (D7) to (D11),

$$R = 0.9039$$

$$k_1 = -1.832$$

$$k_2 = 2.406$$

$$\gamma_1 = 3.688$$

$$\gamma_2 = -0.2687$$

From equations (D1), (D4), and (D6),

$$C_{h_\alpha} = -0.235 + 0.582(0.322) = -0.0473$$

where $C_{h_\alpha}$ is based on $\alpha$ in degrees. For the given configuration, the travel $(x_h')_B$ for angle of attack is obtained when $C_{h_\alpha} \equiv 0$; that is,

$$(x_h')_B = \frac{0.235}{0.322} = 0.73$$

Similarly, the travel $(x_h')_B$ for deflection alone is obtained when $C_{h_\delta} \equiv 0$; that is,

$$(x_h')_B = \frac{0.0508}{0.0857} = 0.593$$

Hence the travel of the hinge line is 13.7 percent of the control-surface root chord.

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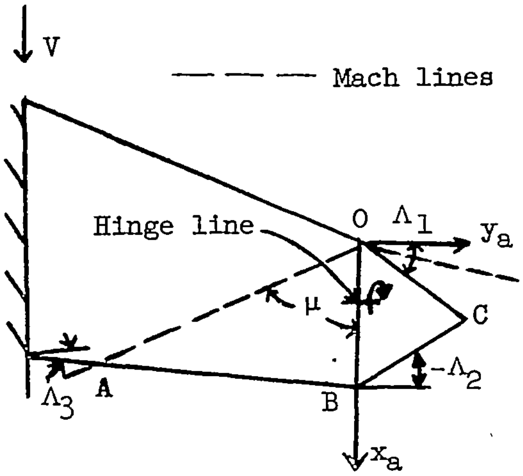
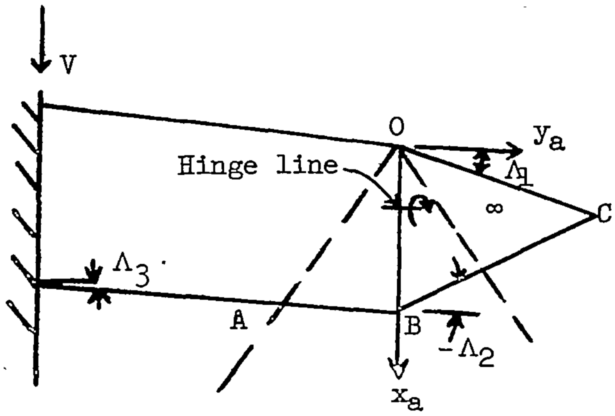
TABLE I.- GENERALIZED FORMULAS FOR $\Delta p/q$ DISTRIBUTIONS FOR A CONTROL SURFACE AT CONSTANT STREAMWISE DEFLECTION ANGLE $\delta$




Subsonic leading edges




Supersonic leading edges

| Region (see sketch) | Formulas for $\Delta p/q$ contributed by constant $\delta$ |
|---|---|
| Subsonic leading edges | |
| OAB, OBC | $$\frac{8(m_1\beta)^{3/2}\delta}{\pi\beta(1 + m_1\beta)}\sqrt{\frac{x_a + \beta y_a}{m_1\beta x_a - \beta y_a}}$$ |
| Supersonic leading edges | |
| $\infty$ | $$\frac{4m_1\delta}{\sqrt{m_1^2\beta^2 - 1}}$$ |
| OAB, OBC | $$\frac{4m_1\delta}{\pi\sqrt{m_1^2\beta^2 - 1}}\cos^{-1}\left(\frac{x_a - m_1\beta^2 y_a}{m_1\beta x_a - \beta y_a}\right)$$ |

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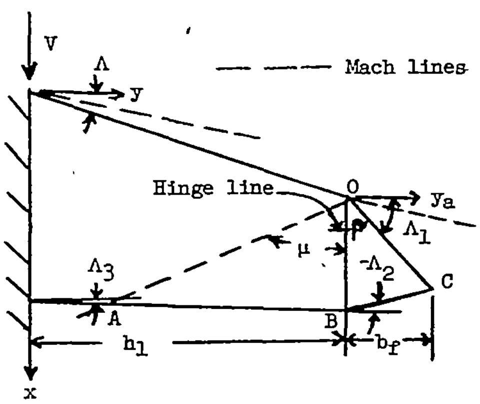
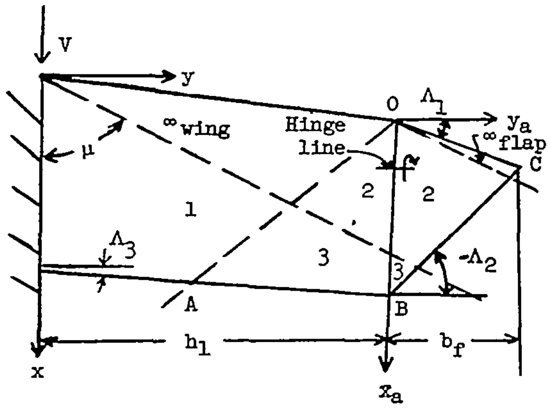
TABLE II.- GENERALIZED FORMULAS FOR $\Delta p/q$ DISTRIBUTIONS FOR A CONTROL SURFACE MOUNTED ON A WING AT CONSTANT STREAMWISE ANGLE OF ATTACK $\alpha$




Subsonic leading edges




Supersonic leading edges

| Region (see sketch) | Formulas for $\Delta p/q$ contributed by constant $\alpha$ |
|---|---|
| | Subsonic leading edges |
| OBC | $\frac{2m\alpha}{E\left(\sqrt{1-m^2\beta^2}\right)}\sqrt{\frac{1+m\beta}{1+m_1\beta}}\left(\sqrt{\frac{m_1x_a-y_a}{mx_a+y_a+2h_1}}+\frac{m_1}{m}\sqrt{\frac{mx_a+y_a+2h_1}{m_1x_a-y_a}}\right)$ |
| | Supersonic leading edges |
| $\infty_{wing}$ | $\frac{4m\alpha}{\sqrt{m^2\beta^2-1}}$ |
| 1 | $\frac{4m\alpha}{\pi\sqrt{m^2\beta^2-1}}\left[\cos^{-1}\left(\frac{x-m\beta^2y}{m\beta x-\beta y}\right)+\cos^{-1}\left(\frac{x+m\beta^2y}{m\beta x+\beta y}\right)\right]$ |
| $\infty_{flap}$ | $\frac{4m_1\alpha}{\sqrt{m_1^2\beta^2-1}}$ |
| 2 | $\frac{4m\alpha}{\pi\sqrt{m^2\beta^2-1}}\cos^{-1}\left(\frac{x_a-m\beta^2y_a}{\beta y_a-m\beta x_a}\right)+\frac{4m_1\alpha}{\pi\sqrt{m_1^2\beta^2-1}}\cos^{-1}\left(\frac{x_a-m_1\beta^2y_a}{m_1\beta x_a-\beta y_a}\right)$ |
| 3 | $\left(\frac{\Delta p}{q}\right)_{region\ 1}+\left(\frac{\Delta p}{q}\right)_{region\ 2}-\left(\frac{\Delta p}{q}\right)_{\infty_{wing}}$ |

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TABLE III.- LOCATION OF FORMULAS FOR THE DEFLECTION CHARACTERISTICS FOR ALL GEOMETRIC CONFIGURATIONS WITHIN THE SCOPE OF THIS PAPER

| $m_2\beta$ (a) | $m_3\beta$ (a) | Formulas |
|---|---|---|
| Subsonic and sonic leading edges (see appendix A) | | |
| [b] $m_2 = m_3 = m_{TE}$ | [b] $m_2 = m_3 = m_{TE}$ | (A5) to (A11) |
| $1 < \lvert m_2\beta \rvert < \infty$ | $1 < \lvert m_3\beta \rvert < \infty$ | (A12) to (A18) |
| $1 < \lvert m_2\beta \rvert < \infty$ | $m_3\beta = \infty$ | (A19) to (A24) |
| $1 < \lvert m_2\beta \rvert < \infty$ | $m_3\beta = 1$ | (A12) to (A18) |
| $m_2\beta = \infty$ | $1 < \lvert m_3\beta \rvert < \infty$ | (A25) to (A29) |
| $m_2\beta = \infty$ | $m_3\beta = 1$ | (A25) to (A29) |
| $m_2\beta = \infty$ | $m_3\beta = \infty$ | (A30) to (A34) |
| $m_2\beta = 1$ | $1 < \lvert m_3\beta \rvert < \infty$ | (A12) to (A18) |
| $m_2\beta = 1$ | $m_3\beta = 1$ | (A12) to (A18) |
| $m_2\beta = 1$ | $m_3\beta = \infty$ | (A35) to (A40) |
| $m_2\beta = -1$ | $1 < \lvert m_3\beta \rvert < \infty$ | (A41) to (A45) |
| $m_2\beta = -1$ | $m_3\beta = 1$ | (A41) to (A45) |
| $m_2\beta = -1$ | $m_3\beta = \infty$ | (A46) to (A50) |
| Supersonic leading edges (see appendix B) | | |
| [b] $m_2 = m_3 = m_{TE}$ | [b] $m_2 = m_3 = m_{TE}$ | (B5) to (B10) |
| $1 < \lvert m_2\beta \rvert < \infty$ | $1 < \lvert m_3\beta \rvert < \infty$ | (B11) to (B15) |
| $1 < \lvert m_2\beta \rvert < \infty$ | $m_3\beta = \infty$ | (B16) to (B20) |
| $1 < \lvert m_2\beta \rvert < \infty$ | $m_3\beta = 1$ | (B21) to (B25) |
| $m_2\beta = \infty$ | $1 < \lvert m_3\beta \rvert < \infty$ | (B26) to (B30) |
| $m_2\beta = \infty$ | $m_3\beta = 1$ | (B31) to (B35) |
| $m_2\beta = \infty$ | $m_3\beta = \infty$ | (B36) to (B40) |
| $m_2\beta = -1$ | $1 < \lvert m_3\beta \rvert < \infty$ | (B41) to (B45) |
| $m_2\beta = -1$ | $m_3\beta = 1$ | (B46) to (B50) |
| $m_2\beta = -1$ | $m_3\beta = \infty$ | (B51) to (B55) |

[a] $m_2\beta$ or $m_3\beta = 1$ refers to sweptback sonic trailing edges; $m_2\beta$ or $m_3\beta = -1$ refers to sweptforward sonic trailing edges; and $m_2\beta$ or $m_3\beta = \infty$ refers to either unswept trailing edges or $M = \infty$.

[b] Basic configuration.

TABLE IV.- CALCULATIONS OF DEFLECTION CHARACTERISTICS FOR UNEQUALLY SWEPT WING AND CONTROL-SURFACE TRAILING EDGES

| $m_1\beta$ | $m_2\beta$ | $m_3\beta$ | $\beta C_{L_\delta}$ | $\beta C_{l_\delta}$ | $\beta C_{m_\delta}$ | $\left(\beta C_{h_\delta}\right)_0$ | $\left(\beta C_{L_\delta}\right)_f$ |
|---|---|---|---|---|---|---|---|
| 0.10 | 2.0 | [a]2.0 | 1.0596 | -0.058868 | -0.67543 | -0.75582 | 1.0949 |
| .10 | 2.0 | 6.0 | 1.1630 | -.097987 | -.76579 | -.75582 | 1.0949 |
| .10 | 2.0 | 16.0 | 1.2054 | -.11687 | -.80595 | -.75582 | 1.0949 |
| .10 | 2.0 | -16.0 | 1.2657 | -.14679 | -.86640 | -.75582 | 1.0949 |
| .10 | 2.0 | -6.0 | 1.3265 | -.18070 | -.93136 | -.75582 | 1.0949 |
| .10 | 2.0 | -2.0 | 1.6406 | -.42404 | -1.3399 | -.75582 | 1.0949 |
| .10 | [a]2.0 | 2.0 | 1.0596 | -.058868 | -.67543 | -.75582 | 1.0949 |
| .10 | 6.0 | 2.0 | 1.0740 | -.064936 | -.67567 | -.72955 | 1.0820 |
| .10 | 16.0 | 2.0 | 1.0785 | -.066870 | -.67594 | -.72177 | 1.0781 |
| .10 | -16.0 | 2.0 | 1.0840 | -.069214 | -.67637 | -.71269 | 1.0735 |
| .10 | -6.0 | 2.0 | 1.0886 | -.071188 | -.67682 | -.70533 | 1.0698 |
| .10 | -2.0 | 2.0 | 1.1036 | -.077623 | -.67878 | -.68295 | 1.0581 |
| .20 | 2.0 | 16.0 | 1.7023 | -.062599 | -1.1718 | -1.5125 | 2.1096 |
| .20 | 6.0 | 16.0 | 1.7355 | -.087745 | -1.1602 | -1.4032 | 2.0568 |
| .20 | [a]16.0 | 16.0 | 1.7464 | -.095880 | -1.1582 | -1.3726 | 2.0414 |
| .20 | -16.0 | 16.0 | 1.7597 | -.10582 | -1.1567 | -1.3379 | 2.0237 |
| .20 | -6.0 | 16.0 | 1.7711 | -.11425 | -1.1561 | -1.3105 | 2.0094 |
| .20 | -2.0 | 16.0 | 1.8088 | -.14217 | -1.1578 | -1.2312 | 1.9667 |
| .20 | 16.0 | 2.0 | 1.5678 | -.020518 | -.99864 | -1.3726 | 2.0414 |
| .20 | 16.0 | 6.0 | 1.6943 | -.071279 | -1.1089 | -1.3726 | 2.0414 |
| .20 | 16.0 | [a]16.0 | 1.7464 | -.095880 | -1.1582 | -1.3726 | 2.0414 |
| .20 | 16.0 | -16.0 | 1.8208 | -.13494 | -1.2328 | -1.3726 | 2.0414 |
| .20 | 16.0 | -6.0 | 1.8961 | -.17932 | -1.3133 | -1.3726 | 2.0414 |
| .20 | 16.0 | -2.0 | 2.2880 | -.49969 | -1.8249 | -1.3726 | 2.0414 |
| .30 | 2.0 | -2.0 | 2.5946 | -.27145 | -2.1092 | -2.2942 | 3.0704 |
| .30 | 6.0 | -2.0 | 2.6851 | -.39421 | -2.1226 | -2.0359 | 2.9484 |
| .30 | 16.0 | -2.0 | 2.7152 | -.43489 | -2.1325 | -1.9681 | 2.9144 |
| .30 | -16.0 | -2.0 | 2.7523 | -.48518 | -2.1473 | -1.8933 | 2.8757 |
| .30 | -6.0 | -2.0 | 2.7840 | -.52835 | -2.1618 | -1.8357 | 2.8452 |
| .30 | [a]-2.0 | -2.0 | 2.8893 | -.67416 | -2.2193 | -1.6766 | 2.7566 |
| .30 | -2.0 | 2.0 | 2.0040 | .020025 | -1.2015 | -1.6766 | 2.7566 |
| .30 | -2.0 | 6.0 | 2.1580 | -.052935 | -1.3356 | -1.6766 | 2.7566 |
| .30 | -2.0 | 16.0 | 2.2218 | -.088390 | -1.3959 | -1.6766 | 2.7566 |
| .30 | -2.0 | -16.0 | 2.3130 | -.14478 | -1.4872 | -1.6766 | 2.7566 |
| .30 | -2.0 | -6.0 | 2.4054 | -.20896 | -1.5861 | -1.6766 | 2.7566 |
| .30 | -2.0 | [a]-2.0 | 2.8893 | -.67416 | -2.2193 | -1.6766 | 2.7566 |
| .40 | -16.0 | 2.0 | 2.3036 | .17878 | -1.4616 | -2.3916 | 3.6479 |
| .40 | -16.0 | 6.0 | 2.4415 | .12003 | -1.5815 | -2.3916 | 3.6479 |
| .40 | -16.0 | 16.0 | 2.4987 | .091428 | -1.6356 | -2.3916 | 3.6479 |
| .40 | -16.0 | [a]-16.0 | 2.5807 | .045879 | -1.7177 | -2.3916 | 3.6479 |
| .40 | -16.0 | -6.0 | 2.6639 | -.0060274 | -1.8068 | -2.3916 | 3.6479 |
| .40 | -16.0 | -2.0 | 3.1012 | -.38350 | -2.3800 | -2.3916 | 3.6479 |
| .40 | 2.0 | -16.0 | 2.5257 | .17564 | -1.8999 | -3.1249 | 3.9980 |
| .40 | 6.0 | -16.0 | 2.5511 | .10014 | -1.7691 | -2.6391 | 3.7744 |
| .40 | 16.0 | -16.0 | 2.5635 | .075753 | -1.7427 | -2.5197 | 3.7146 |
| .40 | [a]-16.0 | -16.0 | 2.5807 | .045879 | -1.7177 | -2.3916 | 3.6479 |
| .40 | -6.0 | -16.0 | 2.5967 | .020436 | -1.7017 | -2.2958 | 3.5962 |
| .40 | -2.0 | -16.0 | 2.6567 | -.064688 | -1.6741 | -2.0420 | 3.4503 |

[a]Basic configuration.

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TABLE IV.- CALCULATIONS OF DEFLECTION CHARACTERISTICS FOR UNEQUALLY SWEPT WING AND CONTROL-SURFACE TRAILING EDGES - Continued

| $m_1\beta$ | $m_2\beta$ | $m_3\beta$ | $\beta C_{L_\delta}$ | $\beta C_{l_\delta}$ | $\beta C_{m_\delta}$ | $(\beta C_{h_\delta})_o$ | $(\beta C_{L_\delta})_f$ |
|---|---|---|---|---|---|---|---|
| 0.60 | -16.0 | 2.0 | 2.8710 | 0.49597 | -1.8227 | -3.2539 | 5.0047 |
| .60 | -16.0 | 6.0 | 3.0054 | .43733 | -1.9393 | -3.2539 | 5.0047 |
| .60 | -16.0 | 16.0 | 3.0613 | .40871 | -1.9921 | -3.2539 | 5.0047 |
| .60 | -16.0 | [a]-16.0 | 3.1416 | .36303 | -2.0726 | -3.2539 | 5.0047 |
| .60 | -16.0 | -6.0 | 3.2233 | .31089 | -2.1601 | -3.2539 | 5.0047 |
| .60 | -16.0 | -2.0 | 3.6549 | -.070052 | -2.7273 | -3.2539 | 5.0047 |
| .60 | 2.0 | -16.0 | 3.2063 | .61142 | -2.6644 | -5.0439 | 5.8217 |
| .60 | 6.0 | -16.0 | 3.1419 | .46244 | -2.2274 | -3.7891 | 5.2789 |
| .60 | 16.0 | -16.0 | 3.1384 | .41727 | -2.1469 | -3.5230 | 5.1466 |
| .60 | [a]-16.0 | -16.0 | 3.1416 | .36303 | -2.0726 | -3.2539 | 5.0047 |
| .60 | -6.0 | -16.0 | 3.1496 | .31751 | -2.0254 | -3.0629 | 4.8983 |
| .60 | -2.0 | -16.0 | 3.2000 | .16755 | -1.9388 | -2.5963 | 4.6138 |
| .70 | -2.0 | 2.0 | 3.0795 | .53050 | -1.7186 | -2.8075 | 5.1072 |
| .70 | -2.0 | 6.0 | 3.2494 | .43365 | -1.8659 | -2.8075 | 5.1072 |
| .70 | -2.0 | 16.0 | 3.3202 | .38632 | -1.9327 | -2.8075 | 5.1072 |
| .70 | -2.0 | -16.0 | 3.4219 | .31076 | -2.0346 | -2.8075 | 5.1072 |
| .70 | -2.0 | -6.0 | 3.5253 | .22444 | -2.1455 | -2.8075 | 5.1072 |
| .70 | -2.0 | [a]-2.0 | 4.0734 | -.40734 | -2.8665 | -2.8075 | 5.1072 |
| .70 | 2.0 | -2.0 | 3.8678 | .72405 | -3.5364 | -6.2033 | 6.7487 |
| .70 | 6.0 | -2.0 | 3.8358 | .37776 | -2.9917 | -4.3482 | 5.9754 |
| .70 | 16.0 | -2.0 | 3.8537 | .26582 | -2.9183 | -3.9863 | 5.7964 |
| .70 | -16.0 | -2.0 | 3.8869 | .12710 | -2.8653 | -3.6311 | 5.6081 |
| .70 | -6.0 | -2.0 | 3.9226 | .0071831 | -2.8436 | -3.3855 | 5.4693 |
| .70 | [a]-2.0 | -2.0 | 4.0734 | -.40734 | -2.8665 | -2.8075 | 5.1072 |
| .80 | 16.0 | 2.0 | 3.3970 | .96318 | -2.2869 | -4.4297 | 6.4109 |
| .80 | 16.0 | 6.0 | 3.5128 | .91656 | -2.3872 | -4.4297 | 6.4109 |
| .80 | 16.0 | [a]16.0 | 3.5611 | .89376 | -2.4328 | -4.4297 | 6.4109 |
| .80 | 16.0 | -16.0 | 3.6305 | .85733 | -2.5024 | -4.4297 | 6.4109 |
| .80 | 16.0 | -6.0 | 3.7012 | .81570 | -2.5782 | -4.4297 | 6.4109 |
| .80 | 16.0 | -2.0 | 4.0762 | .51053 | -3.0719 | -4.4297 | 6.4109 |
| .80 | 2.0 | 16.0 | 3.8749 | 1.2294 | -3.6539 | -7.5618 | 7.7070 |
| .80 | 6.0 | 16.0 | 3.6021 | .96371 | -2.6026 | -4.9032 | 6.6436 |
| .80 | [a]16.0 | 16.0 | 3.5611 | .89376 | -2.4328 | -4.4297 | 6.4109 |
| .80 | -16.0 | 16.0 | 3.5292 | .81377 | -2.2813 | -3.9789 | 6.1708 |
| .80 | -6.0 | 16.0 | 3.5143 | .74921 | -2.1870 | -3.6751 | 5.9966 |
| .80 | -2.0 | 16.0 | 3.5170 | .54718 | -2.0108 | -2.9855 | 5.5532 |
| .90 | 2.0 | [a]2.0 | 4.1779 | 1.6247 | -4.2623 | -9.1921 | 8.7145 |
| .90 | 2.0 | 6.0 | 4.2427 | 1.6096 | -4.3184 | -9.1921 | 8.7145 |
| .90 | 2.0 | 16.0 | 4.2698 | 1.6021 | -4.3440 | -9.1921 | 8.7145 |
| .90 | 2.0 | -16.0 | 4.3087 | 1.5903 | -4.3830 | -9.1921 | 8.7145 |
| .90 | 2.0 | -6.0 | 4.3484 | 1.5767 | -4.4255 | -9.1921 | 8.7145 |
| .90 | 2.0 | -2.0 | 4.5591 | 1.4772 | -4.7031 | -9.1921 | 8.7145 |
| .90 | [a]2.0 | 2.0 | 4.1779 | 1.6247 | -4.2623 | -9.1921 | 8.7145 |
| .90 | 6.0 | 2.0 | 3.7104 | 1.2883 | -2.6991 | -5.4580 | 7.2888 |
| .90 | 16.0 | 2.0 | 3.6318 | 1.2108 | -2.4619 | -4.8565 | 6.9951 |
| .90 | -16.0 | 2.0 | 3.5621 | 1.1270 | -2.2513 | -4.3011 | 6.6980 |
| .90 | -6.0 | 2.0 | 3.5199 | 1.0626 | -2.1193 | -3.9363 | 6.4860 |
| .90 | -2.0 | 2.0 | 3.4517 | .87676 | -1.8602 | -3.1363 | 5.9586 |

[a]Basic configuration.

TABLE IV.- CALCULATIONS OF DEFLECTION CHARACTERISTICS FOR UNEQUALLY SWEPT WING AND CONTROL-SURFACE TRAILING EDGES - Continued

| $m_1\beta$ | $m_2\beta$ | $m_3\beta$ | $\beta C_{L_\delta}$ | $\beta C_{l_\delta}$ | $\beta C_{m_\delta}$ | $(\beta C_{h_\delta})_0$ | $(\beta C_{L_\delta})_f$ |
|---|---|---|---|---|---|---|---|
| 1.0 | 2.0 | [a]2.0 | 4.6188 | 2.0528 | -5.1320 | -11.199 | 9.7930 |
| 1.0 | 2.0 | 6.0 | 4.6758 | 2.0407 | -5.1813 | -11.199 | 9.7930 |
| 1.0 | 2.0 | 16.0 | 4.6996 | 2.0347 | -5.2038 | -11.199 | 9.7930 |
| 1.0 | 2.0 | -16.0 | 4.7338 | 2.0252 | -5.2381 | -11.199 | 9.7930 |
| 1.0 | 2.0 | -6.0 | 4.7687 | 2.0143 | -5.2756 | -11.199 | 9.7930 |
| 1.0 | 2.0 | -2.0 | 4.9544 | 1.9344 | -5.5203 | -11.199 | 9.7930 |
| 1.0 | 6.0 | 2.0 | 3.9618 | 1.5791 | -2.9314 | -6.0162 | 7.9151 |
| 1.0 | 6.0 | [a]6.0 | 4.0567 | 1.5454 | -3.0136 | -6.0162 | 7.9151 |
| 1.0 | 6.0 | 16.0 | 4.0964 | 1.5289 | -3.0510 | -6.0162 | 7.9151 |
| 1.0 | 6.0 | -16.0 | 4.1535 | 1.5025 | -3.1082 | -6.0162 | 7.9151 |
| 1.0 | 6.0 | -6.0 | 4.2117 | 1.4723 | -3.1706 | -6.0162 | 7.9151 |
| 1.0 | 6.0 | -2.0 | 4.5211 | 1.2503 | -3.5785 | -6.0162 | 7.9151 |
| 1.0 | 16.0 | 2.0 | 3.8564 | 1.4781 | -2.6316 | -5.2693 | 7.5531 |
| 1.0 | 16.0 | 6.0 | 3.9632 | 1.4354 | -2.7241 | -5.2693 | 7.5531 |
| 1.0 | 16.0 | [a]16.0 | 4.0078 | 1.4145 | -2.7662 | -5.2693 | 7.5531 |
| 1.0 | 16.0 | -16.0 | 4.0721 | 1.3811 | -2.8305 | -5.2693 | 7.5531 |
| 1.0 | 16.0 | -6.0 | 4.1375 | 1.3429 | -2.9008 | -5.2693 | 7.5531 |
| 1.0 | 16.0 | -2.0 | 4.4857 | 1.0619 | -3.3597 | -5.2693 | 7.5531 |
| 1.0 | -16.0 | 2.0 | 3.7634 | 1.3715 | -2.3731 | -4.6007 | 7.1941 |
| 1.0 | -16.0 | 6.0 | 3.8845 | 1.3167 | -2.4779 | -4.6007 | 7.1941 |
| 1.0 | -16.0 | 16.0 | 3.9350 | 1.2898 | -2.5256 | -4.6007 | 7.1941 |
| 1.0 | -16.0 | [a]-16.0 | 4.0078 | 1.2469 | -2.5985 | -4.6007 | 7.1941 |
| 1.0 | -16.0 | -6.0 | 4.0820 | 1.1978 | -2.6781 | -4.6007 | 7.1941 |
| 1.0 | -16.0 | -2.0 | 4.4766 | .83691 | -3.1982 | -4.6007 | 7.1941 |
| 1.0 | -6.0 | 2.0 | 3.7069 | 1.2912 | -2.2150 | -4.1726 | 6.9421 |
| 1.0 | -6.0 | 6.0 | 3.8399 | 1.2252 | -2.3301 | -4.1726 | 6.9421 |
| 1.0 | -6.0 | 16.0 | 3.8954 | 1.1928 | -2.3825 | -4.1726 | 6.9421 |
| 1.0 | -6.0 | -16.0 | 3.9753 | 1.1410 | -2.4626 | -4.1726 | 6.9421 |
| 1.0 | -6.0 | [a]-6.0 | 4.0567 | 1.0818 | -2.5500 | -4.1726 | 6.9421 |
| 1.0 | -6.0 | -2.0 | 4.4900 | .64672 | -3.1210 | -4.1726 | 6.9421 |
| 1.0 | -2.0 | 2.0 | 3.6119 | 1.0654 | -1.9143 | -3.2643 | 6.3289 |
| 1.0 | -2.0 | 6.0 | 3.7829 | .95619 | -2.0623 | -3.2643 | 6.3289 |
| 1.0 | -2.0 | 16.0 | 3.8543 | .90267 | -2.1297 | -3.2643 | 6.3289 |
| 1.0 | -2.0 | -16.0 | 3.9570 | .81711 | -2.2326 | -3.2643 | 6.3289 |
| 1.0 | -2.0 | -6.0 | 4.0617 | .71922 | -2.3450 | -3.2643 | 6.3289 |
| 1.0 | -2.0 | [a]-2.0 | 4.6188 | 0 | -3.0792 | -3.2643 | 6.3289 |
| 1.75 | 16.0 | 2.0 | 3.9063 | 1.4019 | -2.7510 | -5.7612 | 8.0485 |
| 1.75 | 16.0 | 6.0 | 3.9779 | 1.3863 | -2.8129 | -5.7612 | 8.0485 |
| 1.75 | 16.0 | [a]16.0 | 4.0078 | 1.3786 | -2.8412 | -5.7612 | 8.0485 |
| 1.75 | 16.0 | -16.0 | 4.0510 | 1.3663 | -2.8844 | -5.7612 | 8.0485 |
| 1.75 | 16.0 | -6.0 | 4.0951 | 1.3522 | -2.9317 | -5.7612 | 8.0485 |
| 1.75 | 16.0 | -2.0 | 4.3304 | 1.2484 | -3.2424 | -5.7612 | 8.0485 |
| 1.75 | 2.0 | 16.0 | 4.6330 | 1.6125 | -14.382 | -32.271 | 10.288 |
| 1.75 | 6.0 | 16.0 | 4.0806 | 1.4412 | -3.3224 | -6.9732 | 8.4105 |
| 1.75 | [a]16.0 | 16.0 | 4.0078 | 1.3786 | -2.8412 | -5.7612 | 8.0485 |
| 1.75 | -16.0 | 16.0 | 3.9541 | 1.3019 | -2.4915 | -4.8193 | 7.6894 |
| 1.75 | -6.0 | 16.0 | 3.9302 | 1.2366 | -2.3065 | -4.2736 | 7.4375 |
| 1.75 | -2.0 | 16.0 | 3.9398 | 1.0168 | -2.0292 | -3.2362 | 6.8242 |

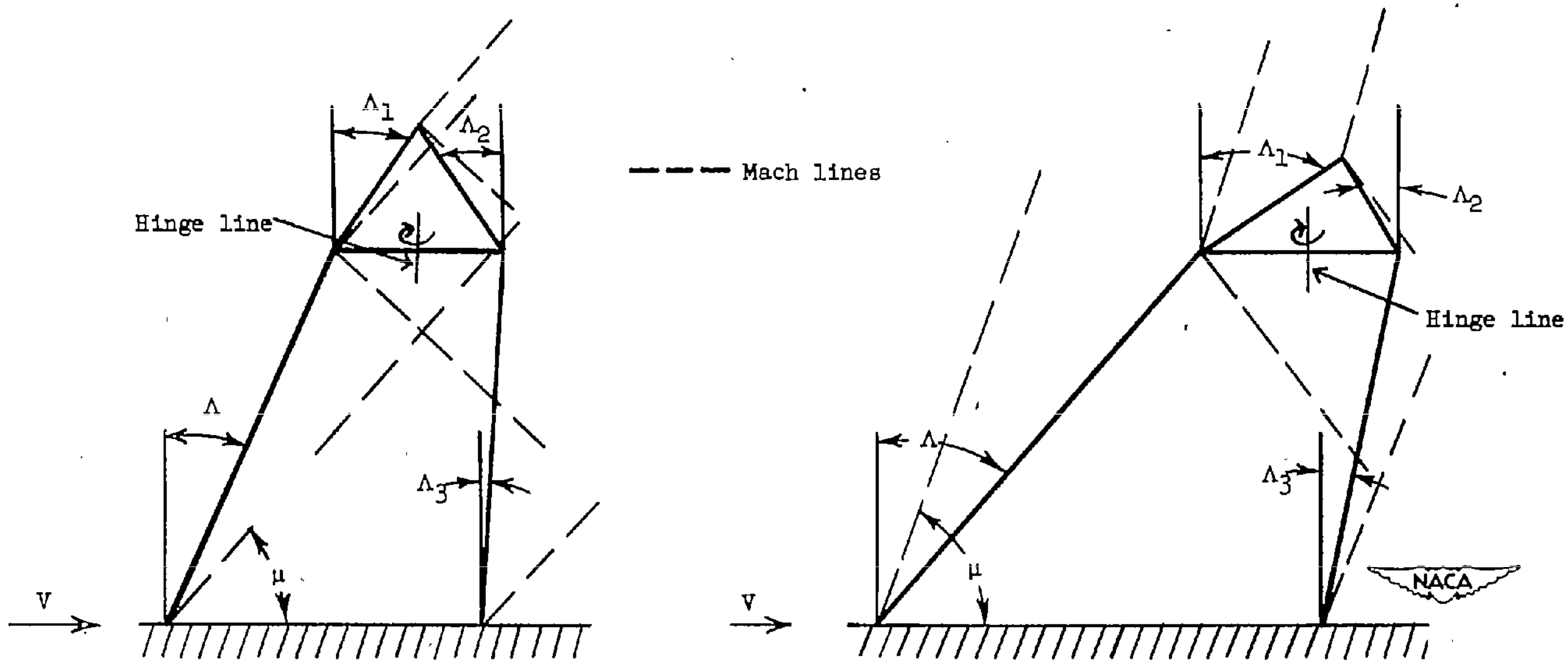
[a]Basic configuration.

NACA

TABLE IV.- CALCULATIONS OF DEFLECTION CHARACTERISTICS FOR UNEQUALLY SWEPT WING AND CONTROL-SURFACE TRAILING EDGES - Concluded

| $m_1\beta$ | $m_2\beta$ | $m_3\beta$ | $\beta C_{L_\delta}$ | $\beta C_{l_\delta}$ | $\beta C_{m_\delta}$ | $(\beta C_{h_\delta})_0$ | $(\beta C_{L_\delta})_f$ |
|---|---|---|---|---|---|---|---|
| 4.0 | -16.0 | 2.0 | 3.8996 | 1.3262 | -2.3102 | -4.8833 | 8.2293 |
| 4.0 | -16.0 | 6.0 | 3.9530 | 1.3190 | -2.3564 | -4.8833 | 8.2293 |
| 4.0 | -16.0 | 16.0 | 3.9755 | 1.3154 | -2.3775 | -4.8833 | 8.2293 |
| 4.0 | -16.0 | [a]-16.0 | 4.0078 | 1.3097 | -2.4099 | -4.8833 | 8.2293 |
| 4.0 | -16.0 | -6.0 | 4.0409 | 1.3032 | -2.4455 | -4.8833 | 8.2293 |
| 4.0 | -16.0 | -2.0 | 4.2182 | 1.2548 | -2.6800 | -4.8833 | 8.2293 |
| | | | | | | | |
| 4.0 | 2.0 | -16.0 | ------ | ------- | ------- | -------- | ------- |
| 4.0 | 6.0 | -16.0 | 4.0713 | 1.3709 | -5.4619 | -12.146 | 8.9503 |
| 4.0 | 16.0 | -16.0 | 4.0273 | 1.3496 | -3.1419 | -6.7458 | 8.5883 |
| 4.0 | [a]-16.0 | -16.0 | 4.0078 | 1.3097 | -2.4099 | -4.8833 | 8.2293 |
| 4.0 | -6.0 | -16.0 | 4.0126 | 1.2673 | -2.1549 | -4.1297 | 7.9773 |
| 4.0 | -2.0 | -16.0 | 4.1138 | 1.0861 | -1.9178 | -3.0212 | 7.3641 |
| | | | | | | | |
| 5.0 | -2.0 | 2.0 | 3.8791 | 1.1930 | -1.6339 | -2.9489 | 7.4681 |
| 5.0 | -2.0 | 6.0 | 4.0031 | 1.1553 | -1.7409 | -2.9489 | 7.4681 |
| 5.0 | -2.0 | 16.0 | 4.0551 | 1.1369 | -1.7900 | -2.9489 | 7.4681 |
| 5.0 | -2.0 | -16.0 | 4.1302 | 1.1072 | -1.8652 | -2.9489 | 7.4681 |
| 5.0 | -2.0 | -6.0 | 4.2069 | 1.0732 | -1.9477 | -2.9489 | 7.4681 |
| 5.0 | -2.0 | [a]-2.0 | 4.6188 | .82112 | -2.4927 | -2.9489 | 7.4681 |
| | | | | | | | |
| 5.0 | 2.0 | -2.0 | ------ | ------- | ------- | -------- | ------- |
| 5.0 | 6.0 | -2.0 | 4.0861 | 1.3592 | -9.5402 | -21.339 | 9.0544 |
| 5.0 | 16.0 | -2.0 | 4.1186 | 1.3353 | -3.4224 | -7.1780 | 8.6924 |
| 5.0 | -16.0 | -2.0 | 4.1911 | 1.2737 | -2.5943 | -4.8399 | 8.3333 |
| 5.0 | -6.0 | -2.0 | 4.2725 | 1.1981 | -2.4140 | -4.0383 | 8.0814 |
| 5.0 | [a]-2.0 | -2.0 | 4.6188 | .82112 | -2.4927 | -2.9489 | 7.4681 |
| | | | | | | | |
| 7.0 | 16.0 | 2.0 | 3.9868 | 1.3435 | -3.6975 | -8.2328 | 8.8200 |
| 7.0 | 16.0 | 6.0 | 4.0016 | 1.3429 | -3.7103 | -8.2328 | 8.8200 |
| 7.0 | 16.0 | [a]16.0 | 4.0078 | 1.3427 | -3.7162 | -8.2328 | 8.8200 |
| 7.0 | 16.0 | -16.0 | 4.0168 | 1.3423 | -3.7252 | -8.2328 | 8.8200 |
| 7.0 | 16.0 | -6.0 | 4.0260 | 1.3418 | -3.7351 | -8.2328 | 8.8200 |
| 7.0 | 16.0 | -2.0 | 4.0754 | 1.3383 | -3.8004 | -8.2328 | 8.8200 |
| | | | | | | | |
| 8.0 | -16.0 | 2.0 | 3.9368 | 1.3267 | -2.1664 | -4.6696 | 8.5030 |
| 8.0 | -16.0 | 6.0 | 3.9718 | 1.3239 | -2.1966 | -4.6696 | 8.5030 |
| 8.0 | -16.0 | 16.0 | 3.9866 | 1.3225 | -2.2105 | -4.6696 | 8.5030 |
| 8.0 | -16.0 | [a]-16.0 | 4.0078 | 1.3202 | -2.2318 | -4.6696 | 8.5030 |
| 8.0 | -16.0 | -6.0 | 4.0296 | 1.3176 | -2.2552 | -4.6696 | 8.5030 |
| 8.0 | -16.0 | -2.0 | 4.1464 | 1.2985 | -2.4098 | -4.6696 | 8.5030 |
| | | | | | | | |
| 9.0 | -2.0 | 2.0 | 3.9283 | 1.2163 | -1.5306 | -2.7703 | 7.6712 |
| 9.0 | -2.0 | 6.0 | 4.0438 | 1.1856 | -1.6302 | -2.7703 | 7.6712 |
| 9.0 | -2.0 | 16.0 | 4.0923 | 1.1705 | -1.6760 | -2.7703 | 7.6712 |
| 9.0 | -2.0 | -16.0 | 4.1623 | 1.1463 | -1.7461 | -2.7703 | 7.6712 |
| 9.0 | -2.0 | -6.0 | 4.2339 | 1.1185 | -1.8231 | -2.7703 | 7.6712 |
| 9.0 | -2.0 | [a]-2.0 | 4.6188 | .91235 | -2.3327 | -2.7703 | 7.6712 |
| | | | | | | | |
| 9.0 | 16.0 | -2.0 | 4.0497 | 1.3384 | -4.4470 | -9.8086 | 8.8954 |
| 9.0 | -16.0 | -2.0 | 4.1375 | 1.3025 | -2.3628 | -4.6123 | 8.5364 |
| 9.0 | -6.0 | -2.0 | 4.2317 | 1.2453 | -2.1634 | -3.7412 | 8.2844 |
| 9.0 | [a]-2.0 | -2.0 | 4.6188 | .91235 | -2.3327 | -2.7703 | 7.6712 |

[a]Basic configuration.




(a) Supersonic leading edges.

(b) Subsonic leading edges.

Figure 1.- General configurations within the scope of the present paper.

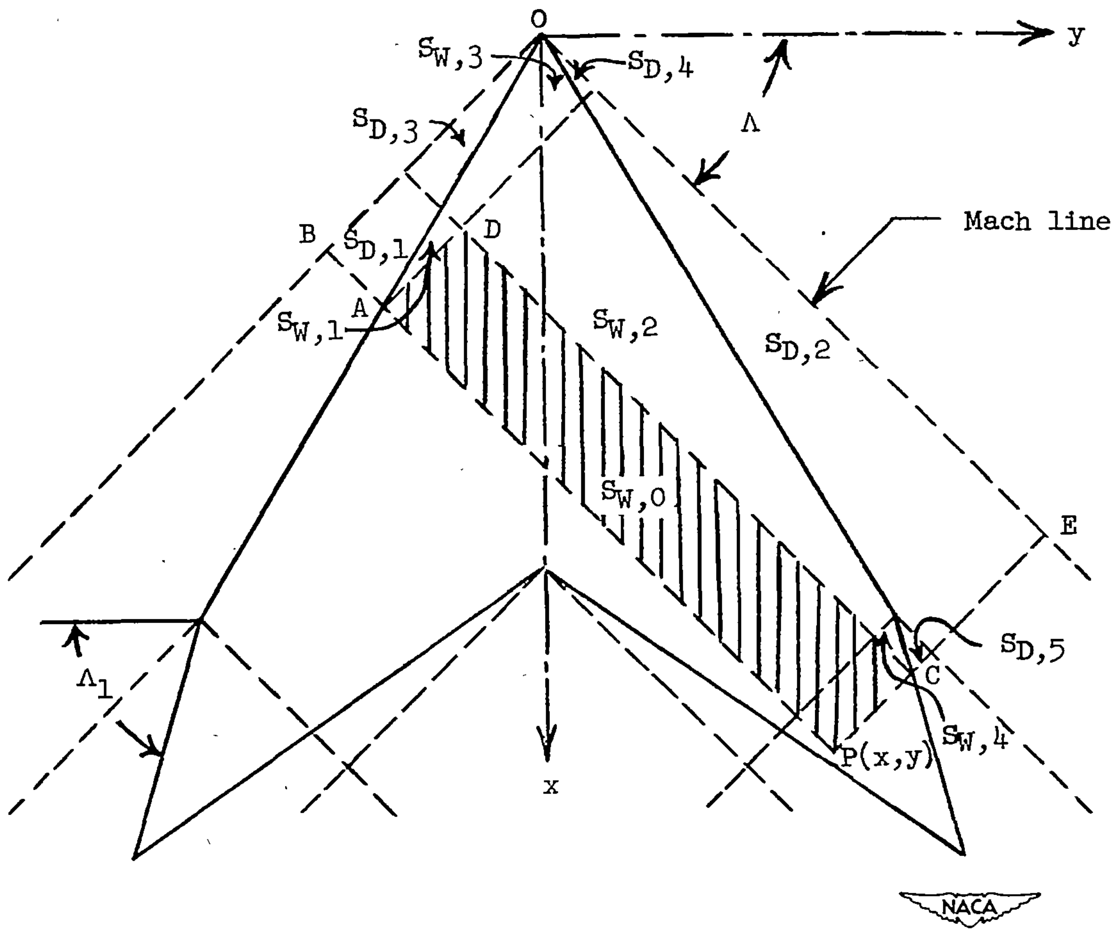
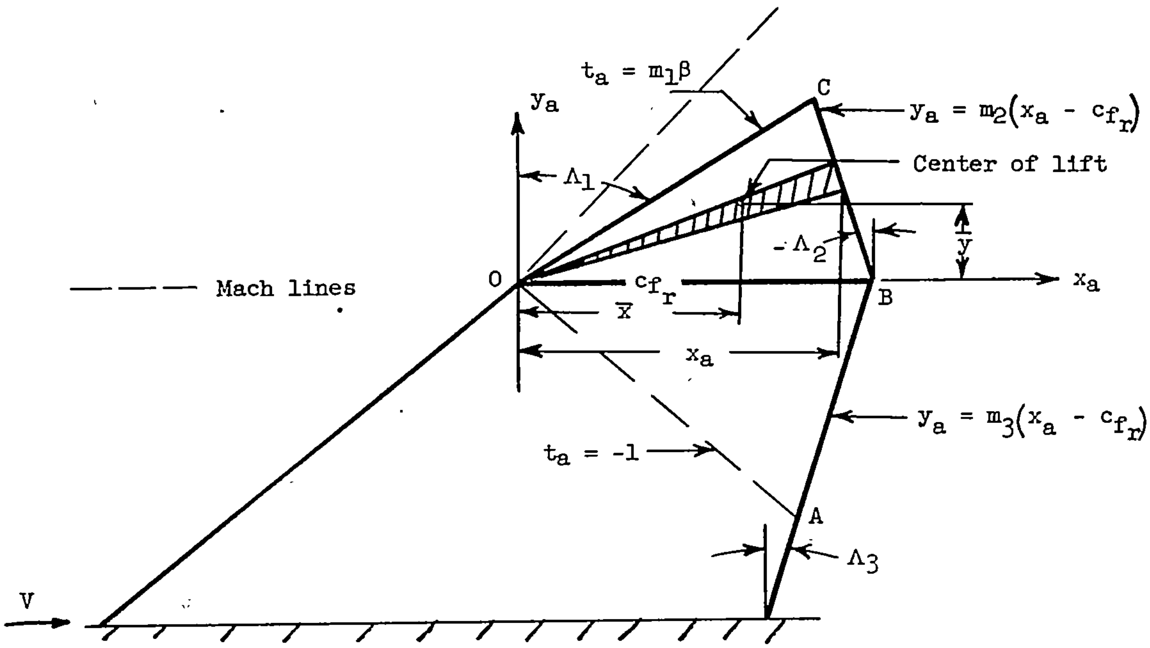



Figure 2.- Sketch of wing regions $S_{W(0,\ldots 4)}$ and external flow regions $S_{D(1,\ldots 5)}$.




$$t_a = \frac{\beta y_a}{x_a} \qquad dS = \frac{x_a^2}{2\beta}\,dt_a$$

$$\bar{x} = \frac{2x}{3} \qquad \bar{y} = \frac{\bar{x}t_a}{\beta}$$

$$\bar{x}' = \bar{x} - x_h$$

Region OBC

$$y_a = m_2(x_a - c_{f_r})$$

$$x_a = \frac{m_2\beta c_{f_r}}{m_2\beta - t_a}$$

$$dS = \frac{m_2^2\beta c_{f_r}^2}{2}\frac{dt}{(m_2\beta - t_a)^2}$$

$$\bar{x} = \frac{2m_2\beta c_{f_r}}{3(m_2\beta - t_a)}$$

$$\bar{y} = \frac{2m_2 c_{f_r} t_a}{3(m_2\beta - t_a)}$$

Region OAB

$$y_a = m_3(x_a - c_{f_r})$$

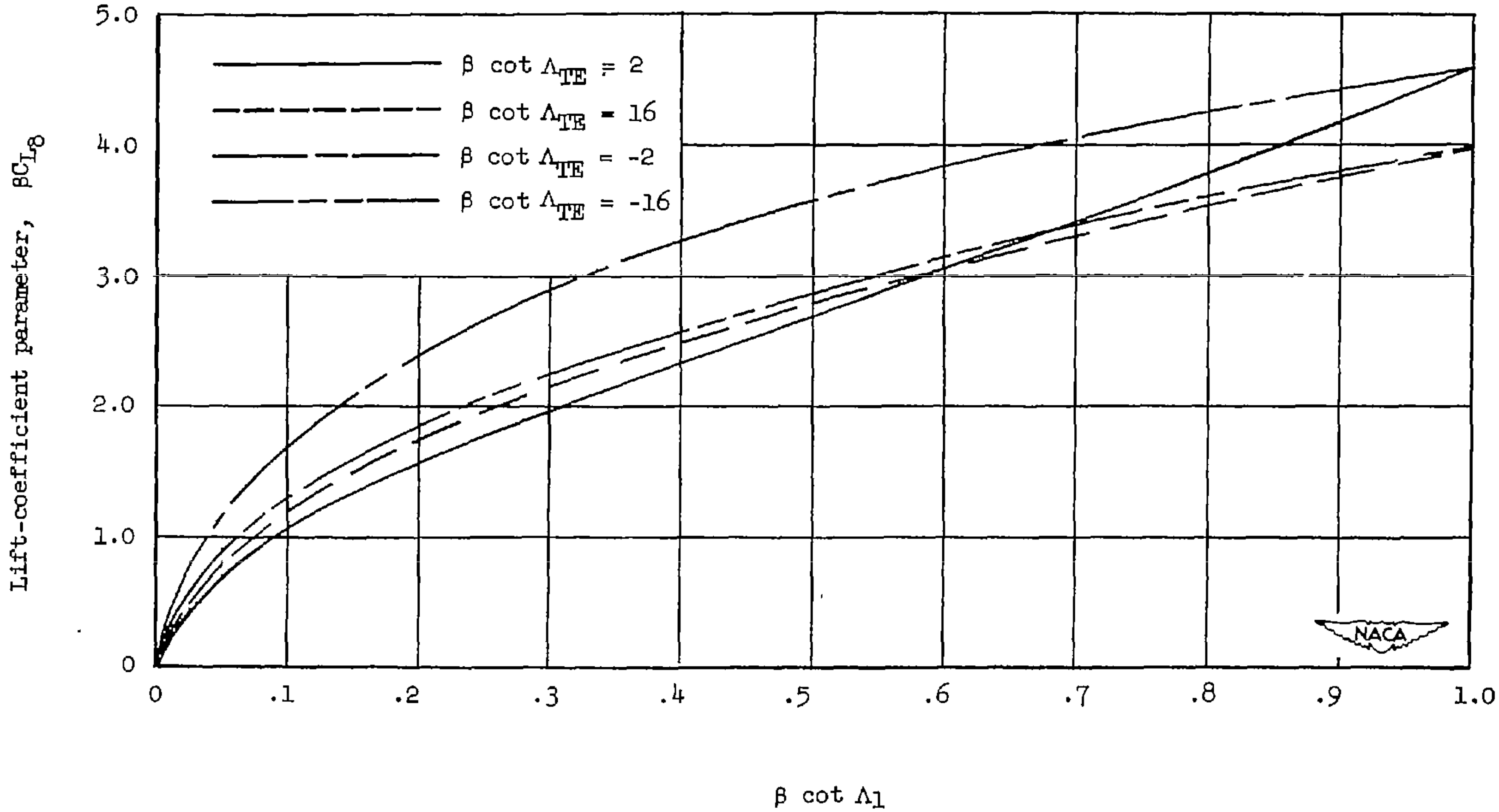
$$x_a = \frac{m_3\beta c_{f_r}}{m_3\beta - t_a}$$

$$dS = \frac{m_3^2\beta c_{f_r}^2}{2}\frac{dt_a}{(m_3\beta - t_a)^2}$$

$$\bar{x} = \frac{2m_3\beta c_{f_r}}{3(m_3\beta - t_a)}$$

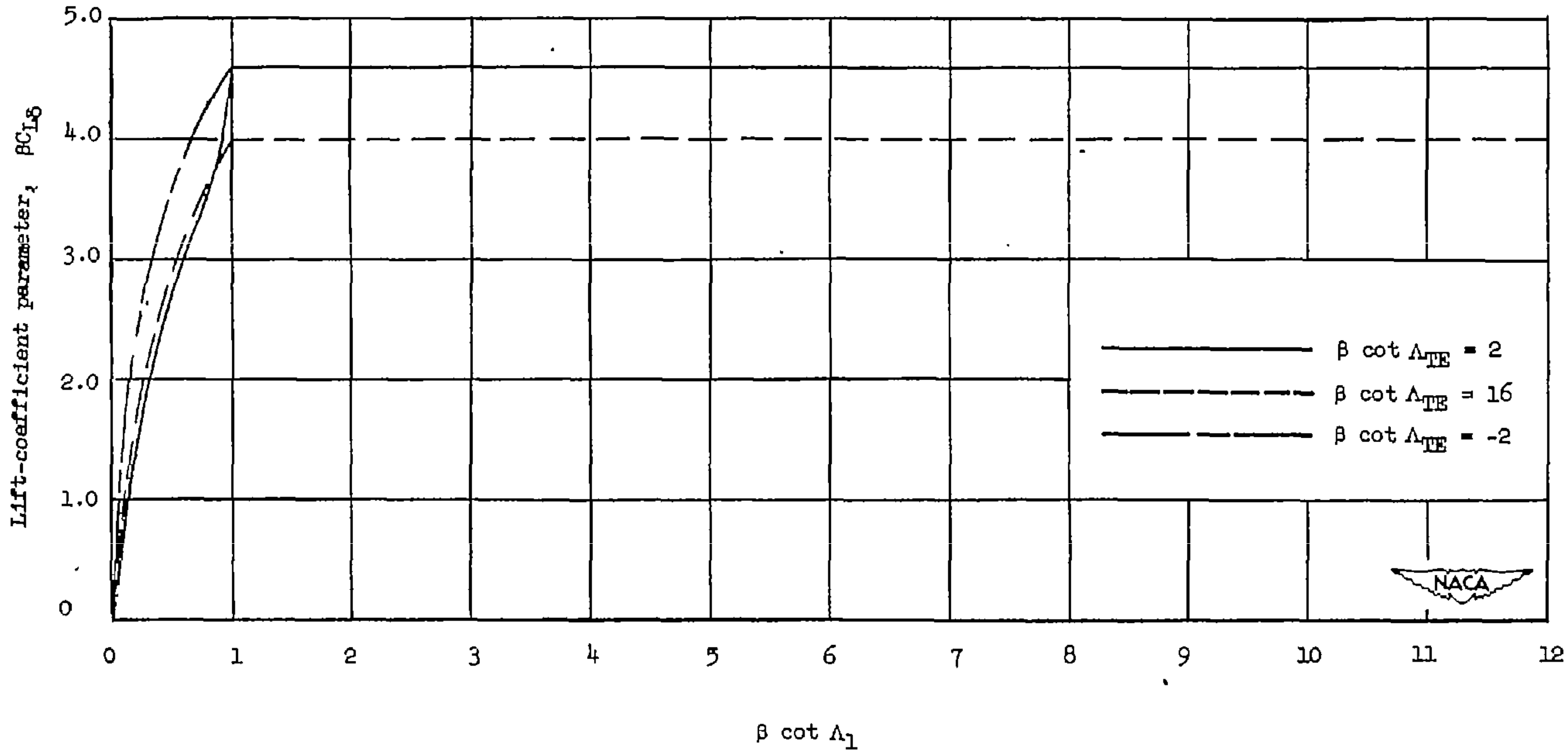
$$\bar{y} = \frac{2m_3 c_{f_r} t_a}{3(m_3\beta - t_a)}.$$

Figure 3.- Illustrative data for polar integration.



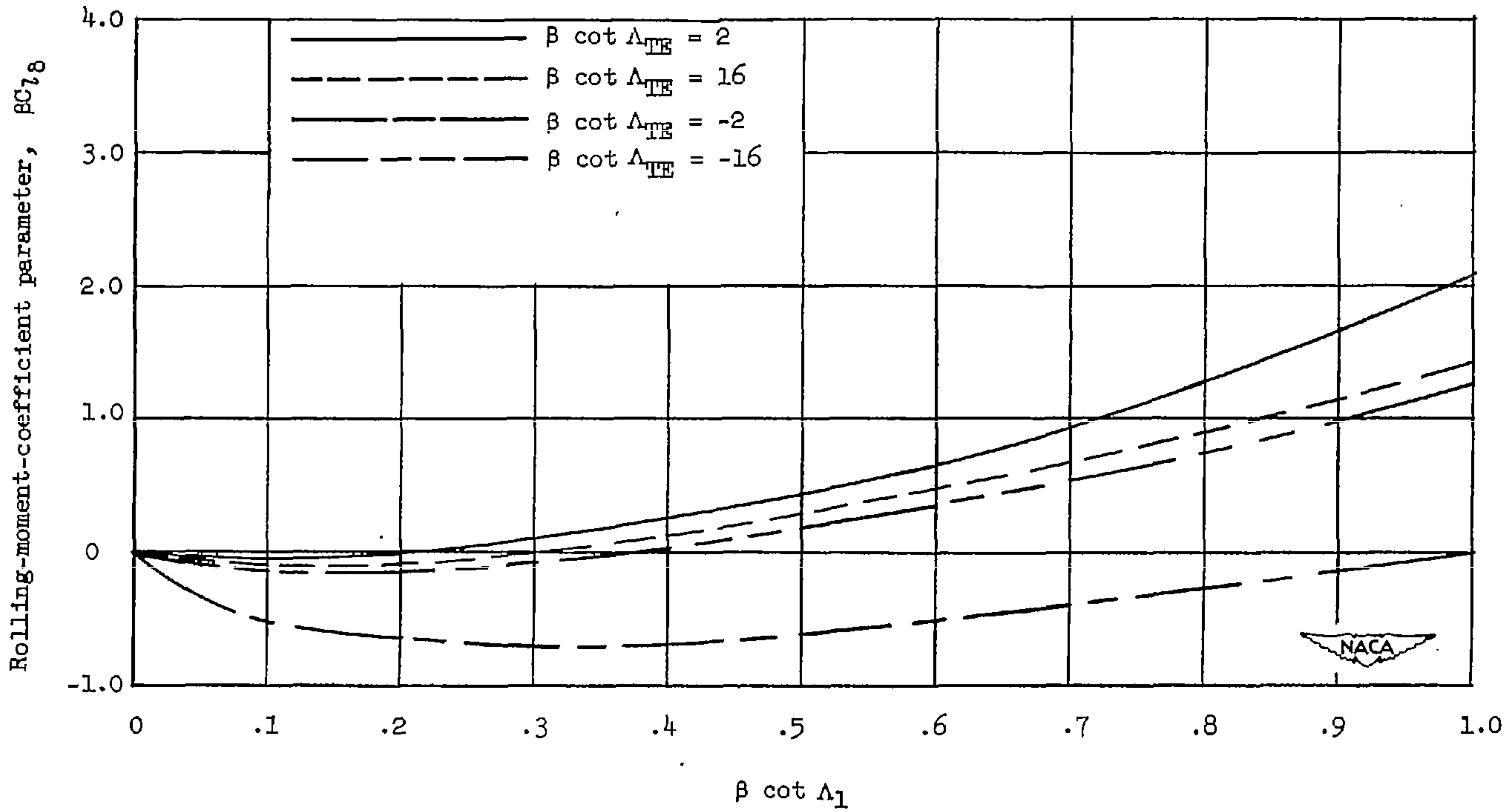

(a) Subsonic leading edges.

Figure 4.- Variation of $\beta C_{L_\delta}$ with $\beta \cot \Lambda_1$ and $\beta \cot \Lambda_{TE}$ for the basic configuration.



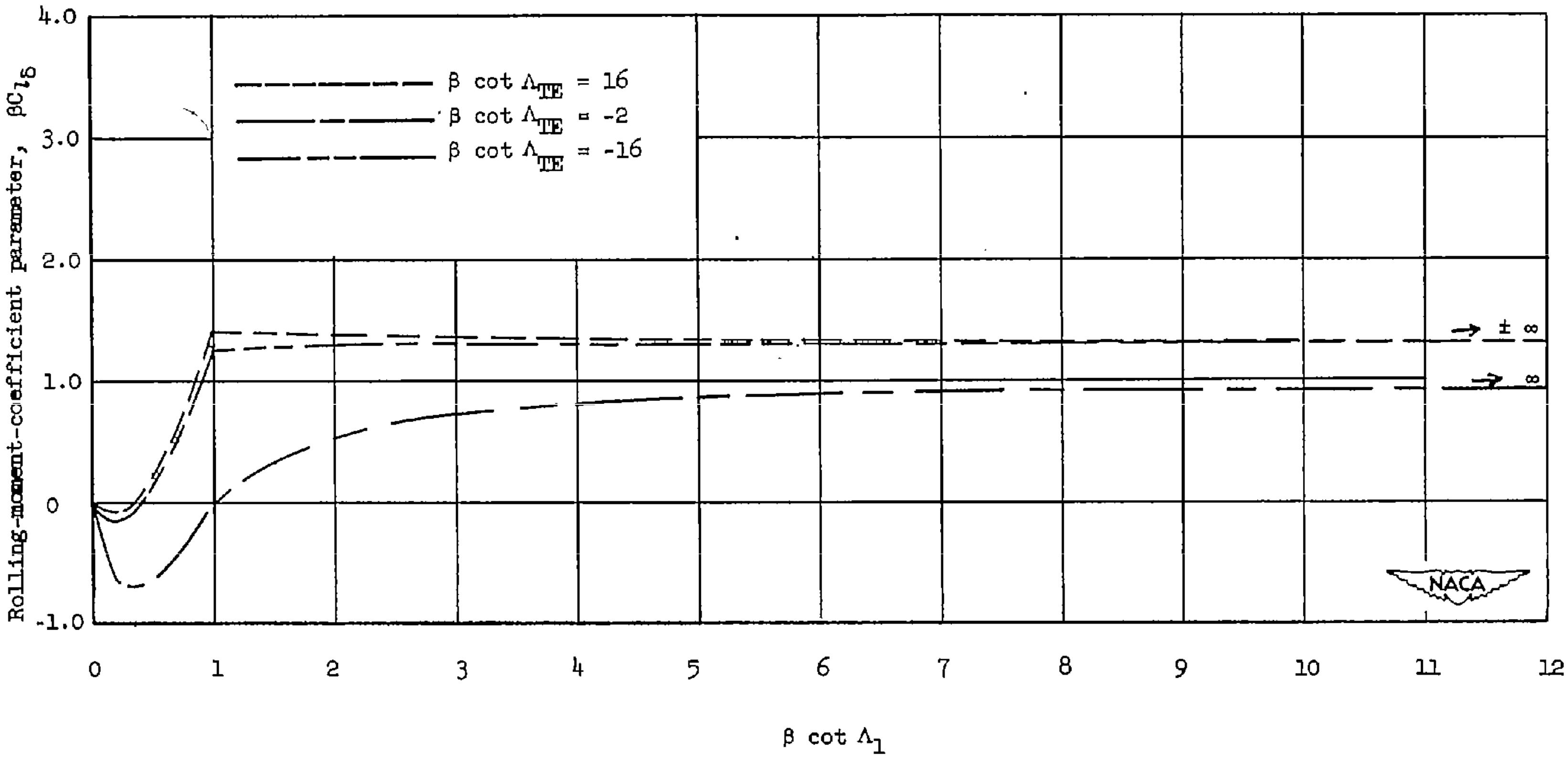

(b) Subsonic and supersonic leading edges.

Figure 4.- Concluded.



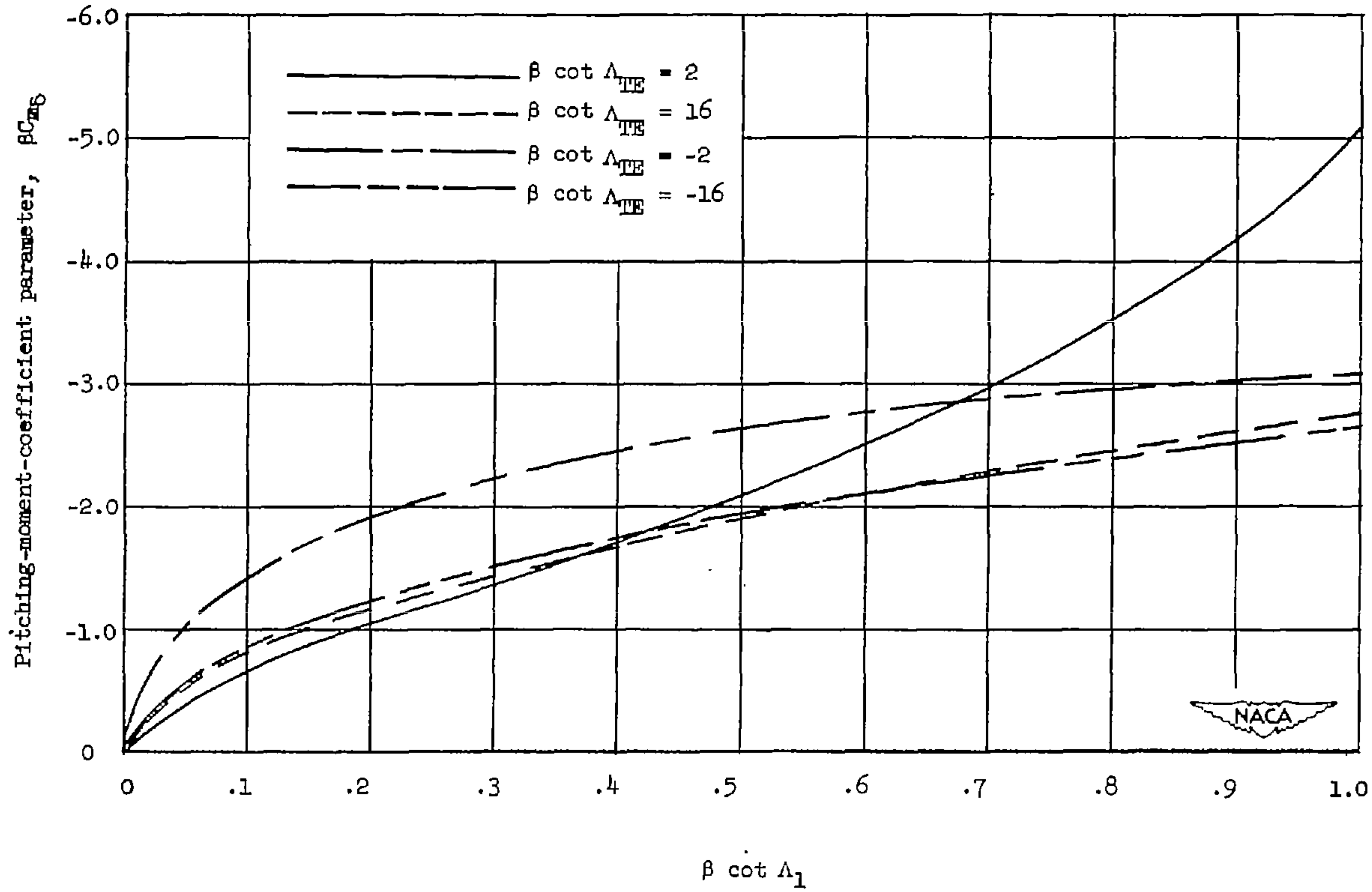

(a) Subsonic leading edges.

Figure 5.- Variation of $\beta C_{l_\delta}$ with $\beta \cot \Lambda_1$ and $\beta \cot \Lambda_{TE}$ for the basic configuration.



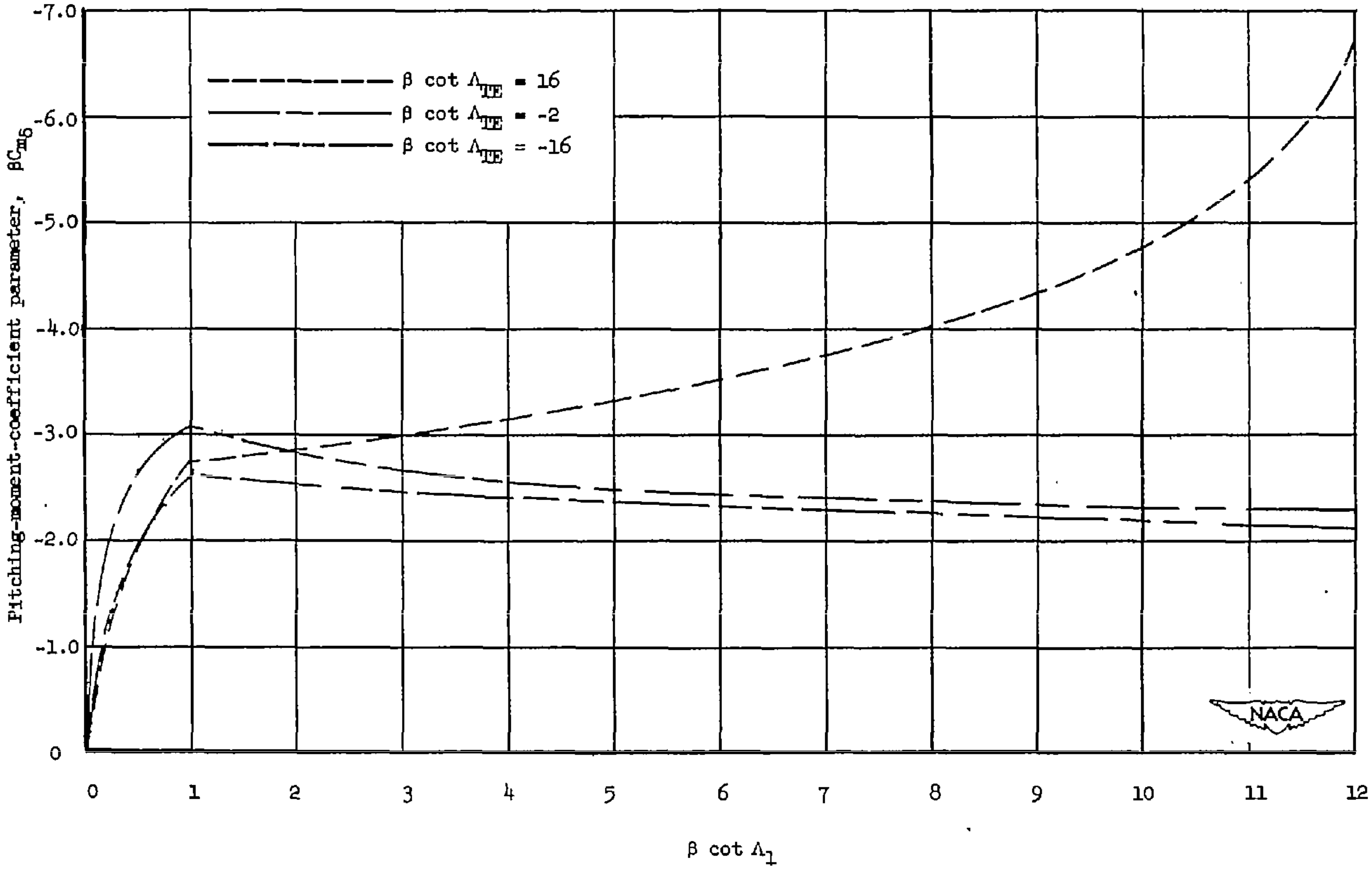

(b) Subsonic and supersonic leading edges.

Figure 5.- Concluded.



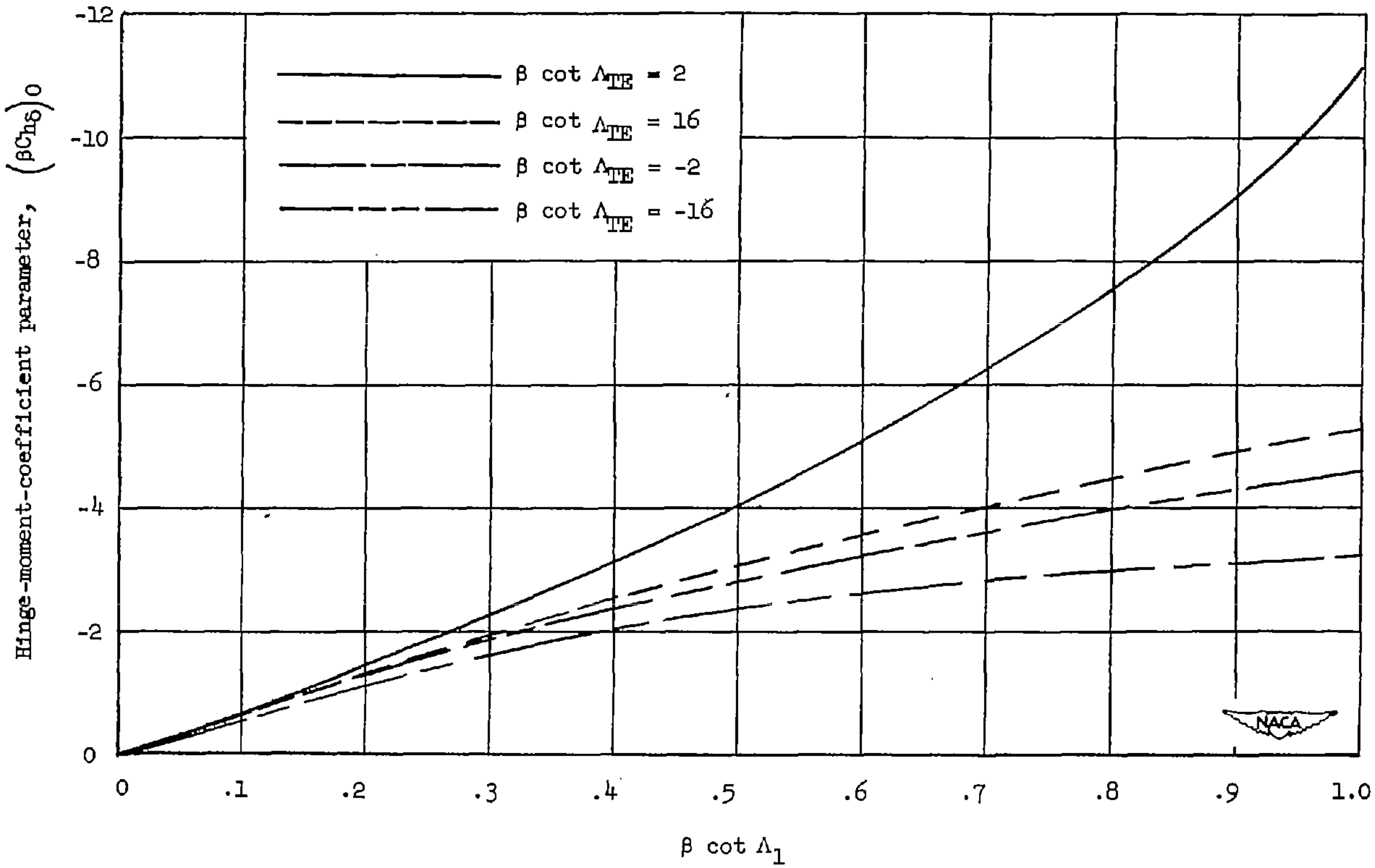

(a) Subsonic leading edges.

Figure 6.- Variation of $\beta C_{m_\delta}$ with $\beta \cot \Lambda_1$ and $\beta \cot \Lambda_{TE}$ for the basic configuration.



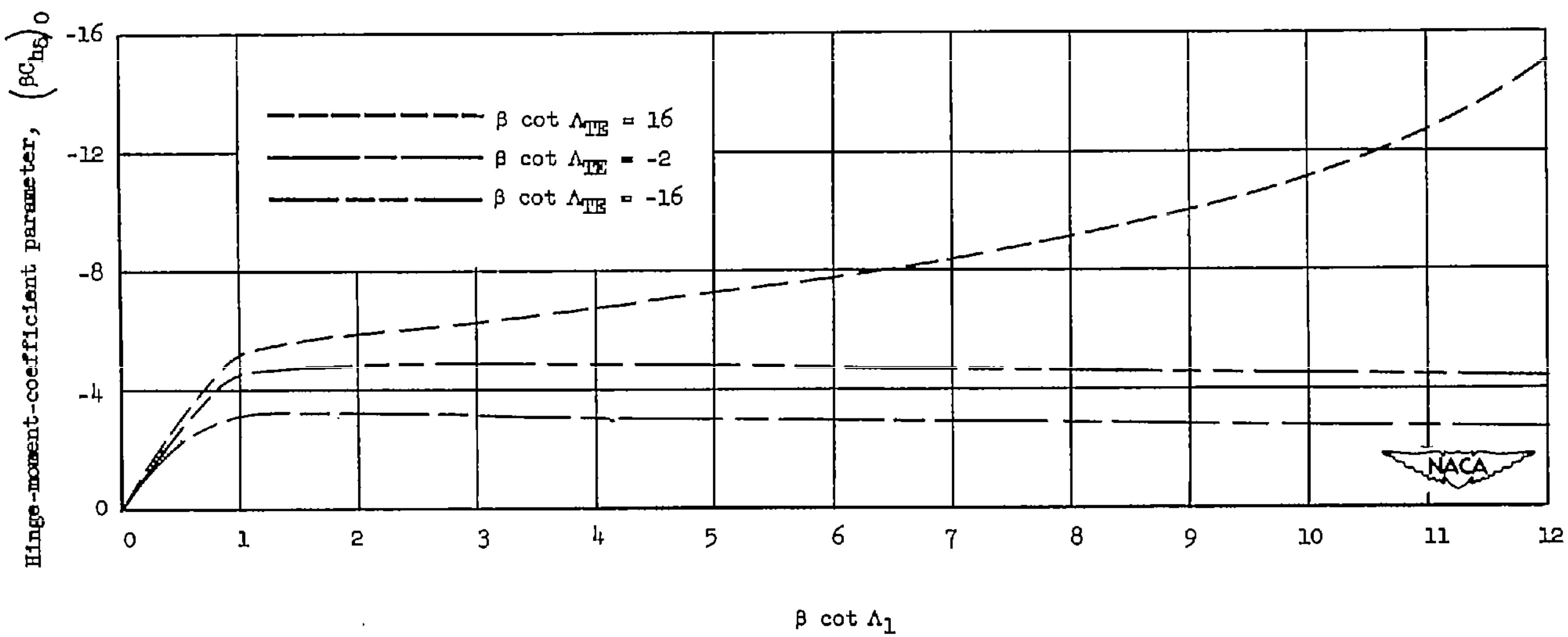

(b) Subsonic and supersonic leading edges.

Figure 6.- Concluded.



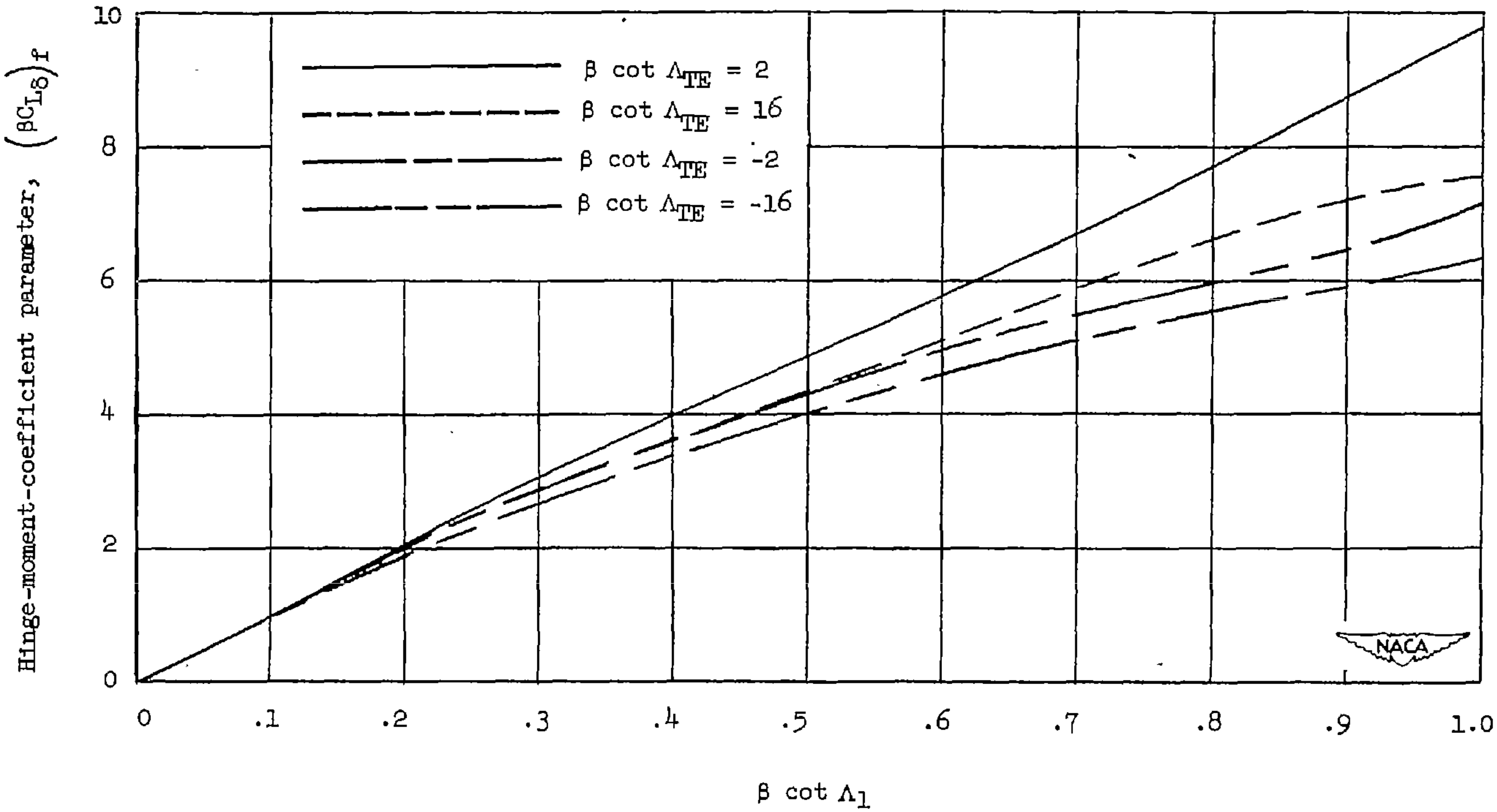

(a) Subsonic leading edges.

Figure 7.- Variation of $\left(\beta C_{h_\delta}\right)_0$ with $\beta \cot \Lambda_1$ and $\beta \cot \Lambda_{TE}$ for the basic configuration.



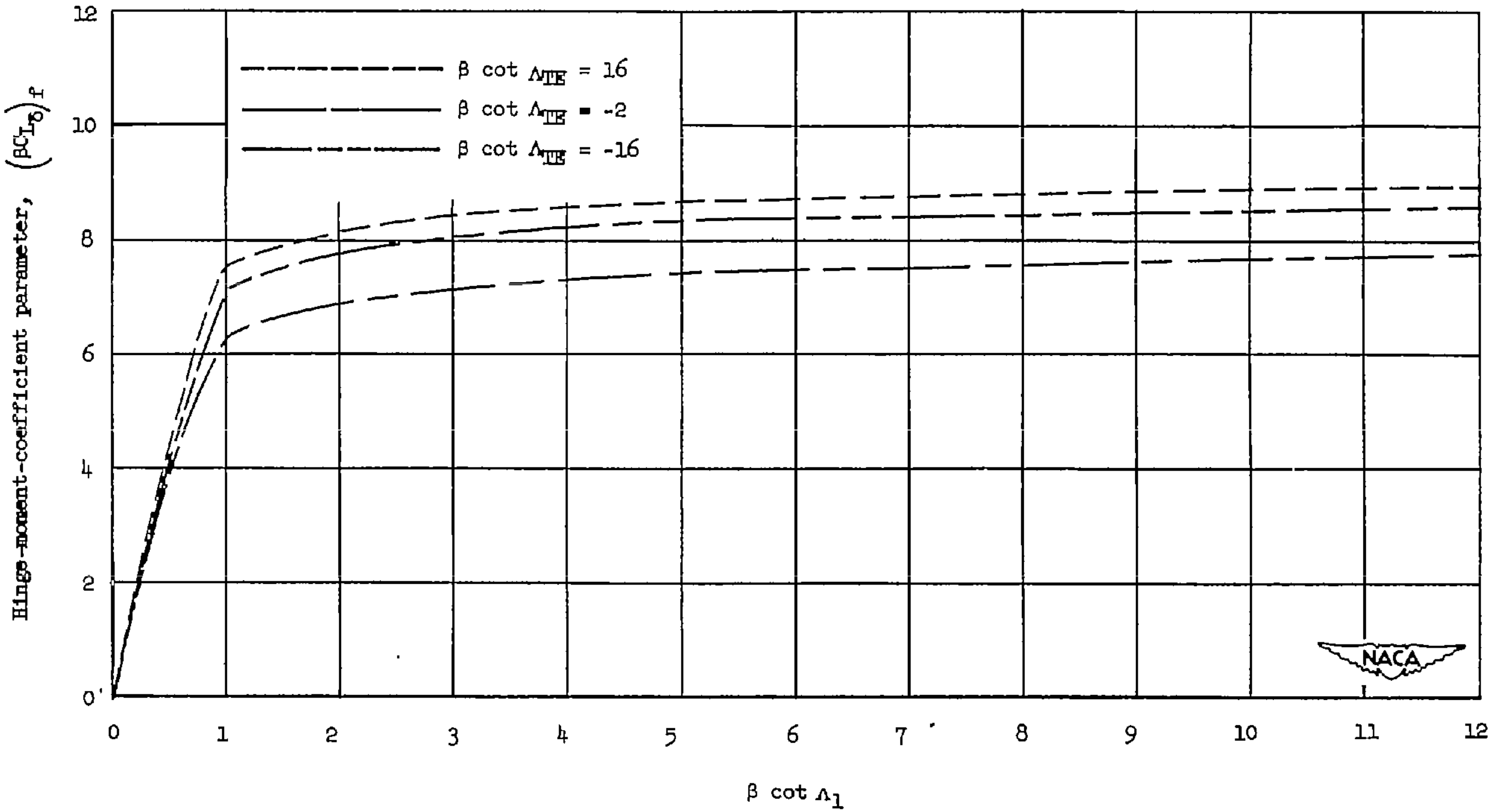

(b) Subsonic and supersonic leading edges.

Figure 7.- Concluded.



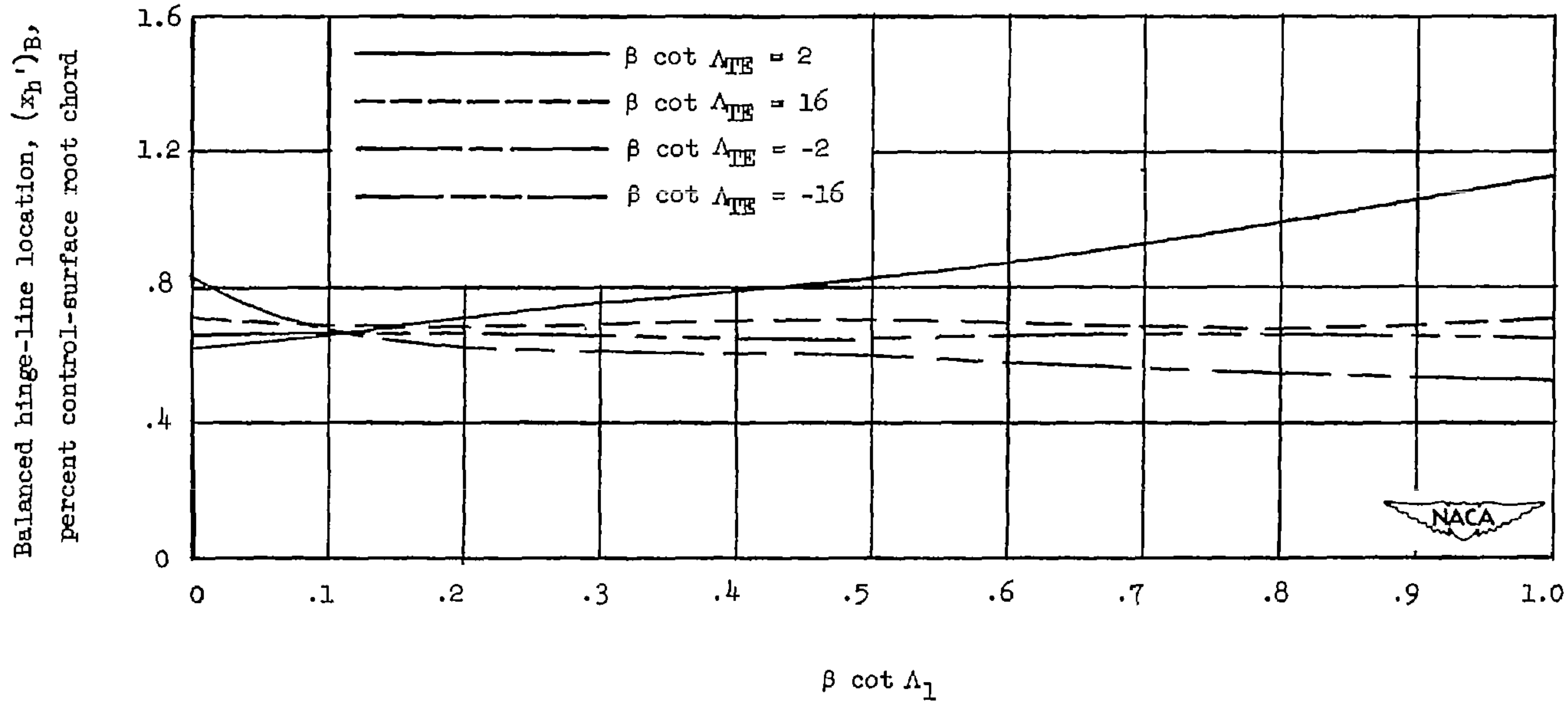

(a) Subsonic leading edges.

Figure 8.- Variation of $(\beta C_{L_\delta})_f$ with $\beta \cot \Lambda_1$ and $\beta \cot \Lambda_{TE}$ for the basic configuration.



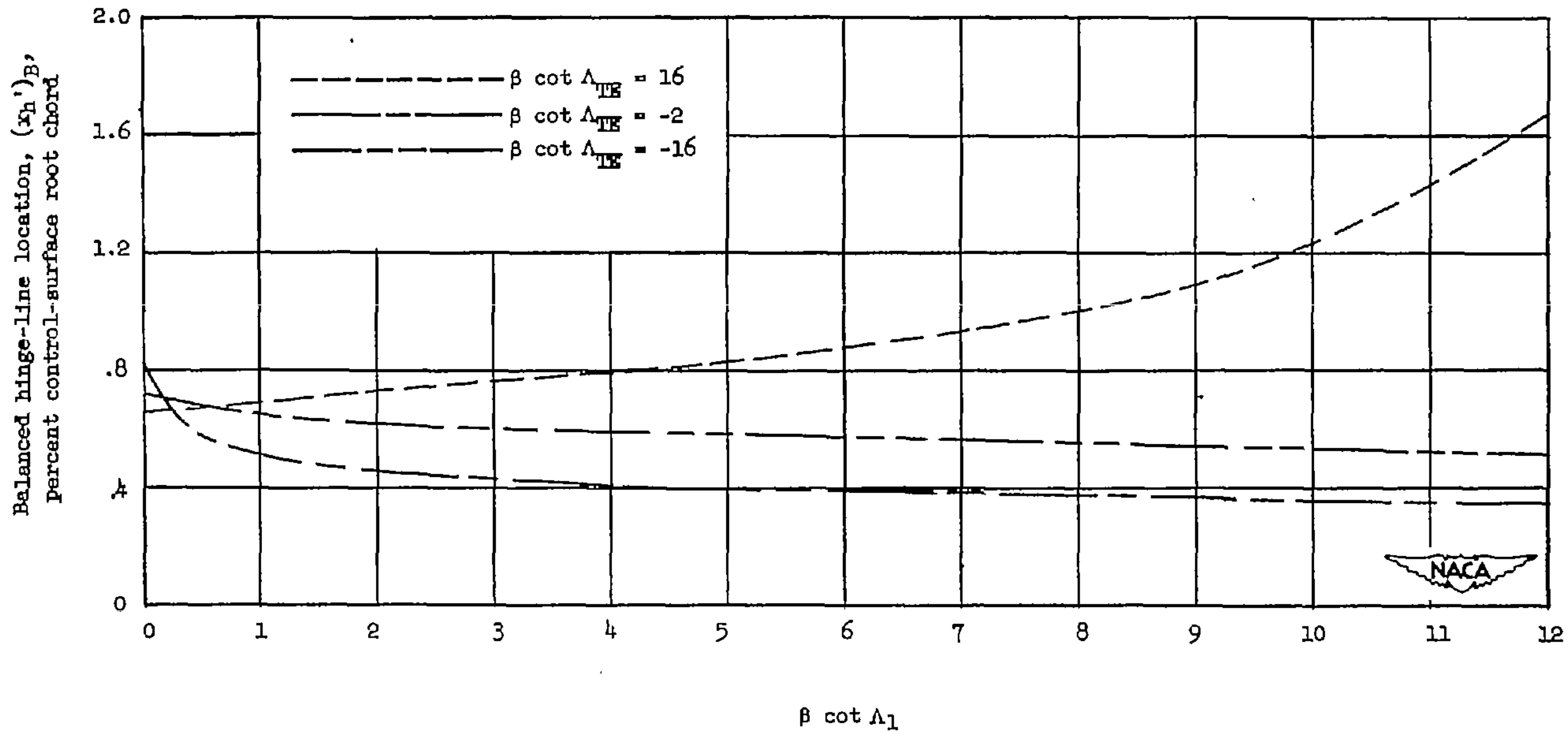

(b) Subsonic and supersonic leading edges.

Figure 8.- Concluded.




(a) Subsonic leading edges.

Figure 9.- Variation of $(x_h')_B$ with $\beta \cot \Lambda_1$ and $\beta \cot \Lambda_{TE}$ for the basic configuration.




(b) Subsonic and supersonic leading edges.

Figure 9.- Concluded.

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